Section 4.2

1. $f(x) = x^2+1$ (a) $f(0) = 0^2+1 = 1$ $f(-1) = (-1)^2+1 = 2$ $f(1/2) = (\frac{1}{2})^2+1 = 1+\frac{1}{4} = \frac{5}{4}$

$f(\sqrt{2}) = (\sqrt{2})^2+1 = 2+1 = 3$ (b) i) $f(x) = x^2+1$ $f(-x) = (-x)^2+1 = x^2+1$ $\Rightarrow f(x) = f(-x)$ for all x

ii) $f(x+1) = (x+1)^2+1 = x^2+2x+1+1 = x^2+2x+2$ $f(x)+f(1) = x^2+1+1^2+1 = x^2+3$

$f(x+1) = f(x)+f(1) \Leftrightarrow x^2+2x+2 = x^2+3 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

iii) $f(2x) = (2x)^2+1 = 4x^2+1$ $2f(x) = 2(x^2+1) = 2x^2+2$ $f(2x) = 2f(x) \Leftrightarrow 4x^2+1 = 2x^2+2$

$\Leftrightarrow 2x^2 = 1 \Leftrightarrow x = \pm\frac{1}{\sqrt{2}}$

4. $f(x) = \frac{x}{1+x^2}$ (a) Compute $f(-\frac{1}{10}) = \frac{-\frac{1}{10}}{1+(\frac{1}{10})^2} = \frac{-\frac{1}{10}}{1+\frac{1}{100}} = -\frac{1}{10}\cdot\frac{100}{101} = -\frac{10}{101}$

$f(0) = 0$ $f(\frac{1}{\sqrt{2}}) = \frac{\frac{1}{\sqrt{2}}}{1+(\frac{1}{\sqrt{2}})^2} = \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{2}} = \frac{\frac{1}{\sqrt{2}}}{3/2} = \frac{1}{\sqrt{2}}\cdot\frac{2}{3} = \frac{\sqrt{2}}{3}$

$f(\sqrt{\pi}) = \frac{\sqrt{\pi}}{1+\pi}$ $f(2) = \frac{2}{1+4} = \frac{2}{5}$

(b) $f(-x) = \frac{-x}{1+(-x)^2} = \frac{-x}{1+x^2} = -f(x)$ $f(\frac{1}{x}) = \frac{\frac{1}{x}}{1+\frac{1}{x^2}} = \frac{x^2\cdot\frac{1}{x}}{1+x^2} = \frac{x}{1+x^2}$

6. $C(x) = 1000+300x+x^2$ (a) $C(0) = 1000$ $C(100) = 1000+300\cdot100+100^2 = 41000$

$C(101)-C(100) = 1000+300\cdot101+101^2-1000-300\cdot100-100^2 = 300(101-100)+(101^2-100^2) =$

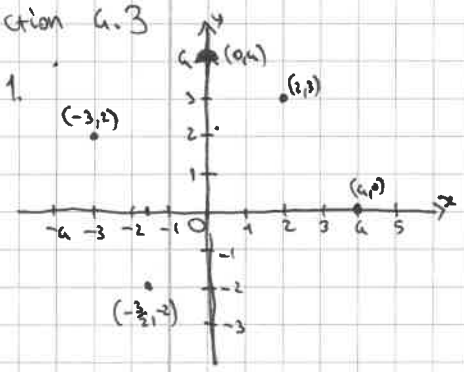
$= 300+(101-100)(101+100) = 300+201 = 501$

$C(x+1)-C(x) = 1000+300(x+1)+(x+1)^2-1000-300x-x^2 =$

$= 300+x^2+2x+1-x^2 = 301+2x$

Section 4.3

1.



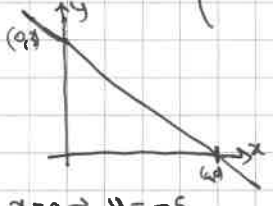
Section 4.4

2. (a) $3x+4y = 12$ we find two points for which the line passes through:

$x=0 \rightarrow 4y = 12 \Rightarrow y = 3$

$y=0 \rightarrow 3x = 12 \Rightarrow x = 4$

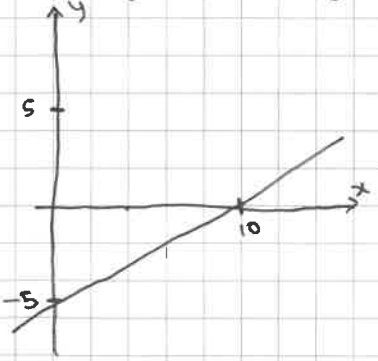
$(0,3)$ and $(4,0)$



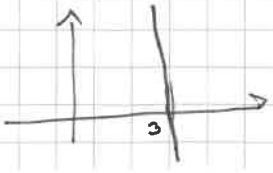
(b) $\frac{x}{10}-\frac{y}{5} = 1$

$x=0 \rightarrow y=-5$

$y=0 \rightarrow x=10$



(c) $x=3$



3. $D(P) = aP + b$ $\quad D(10) = 200$, $D(15) = 150$

let's call $y = D(P)$, $x = P$ we have two points $(10, 200)$ and $(15, 150)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \Rightarrow \frac{y-200}{150-200} = \frac{x-10}{15-10} \Rightarrow \frac{y-200}{-50} = \frac{x-10}{5} \Rightarrow y-200 = -\frac{50}{5}(x-10)$$

$\Rightarrow y = 200 + 10(x-10) = 200 + 10x - 100 = 10x + 100 \qquad D(P) = 10P + 100$

Section 4.5

4. $C(y) = ay + b \qquad C(1000) = 900 \qquad C(y+100) = C(y) + 80 \Rightarrow C(1100) = 980$

$$\begin{cases} a \cdot 1000 + b = 900 \\ a \cdot 1100 + b = 980 \end{cases} \Rightarrow \quad b = 900 - 1000a$$

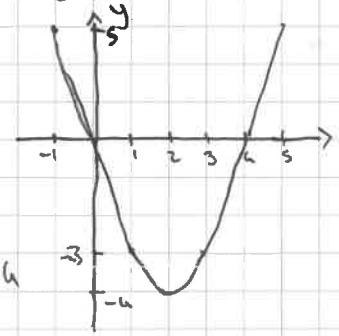
$$1100a + 900 - 1000a = 980 \Rightarrow 100a = 80 \Rightarrow a = \frac{80}{100} = \frac{4}{5}$$

$$b = 900 - 1000 \cdot \frac{4}{5} = 900 - 800 = 100 \Rightarrow C(y) = \frac{4}{5}y + 100$$

Section 4.6

1. $f(x) = x^2 - 4x$

x	-1	0	1	2	3	4	5
$f(x)$	5	0	-3	-4	-3	0	5



(b) min point is for $x = -\frac{b}{2a} = -\frac{-4}{2} = 2$

(c) $f(x) = 0 \Leftrightarrow x^2 - 4x = 0 \Leftrightarrow x(x-4) = 0 \Rightarrow x = 0,\ x = 4$

6. $U(x) = 72 - (4+x)^2 - (4-rx)^2 =$

$= 72 - 16 - 8x - x^2 - 16 + 8rx - r^2x^2 = -(1+r^2)x^2 + 8(r-1)x + 40$

max for $x = -\frac{b}{2a} = -\frac{8(r-1)}{2(-(1+r^2))} = \frac{4r-4}{1+r^2}$