Section 4.7

2. (a) $x^2+x-2 \Rightarrow x=\dfrac{-1\pm\sqrt{1+2\cdot 4}}{2} = \dfrac{-1\pm 3}{2} = \left\langle \begin{array}{l} -\frac{4}{2}=-2 \\ \frac{2}{2}=1 \end{array}\right]$ both integer solutions

(b) $x^3-x^2-25x+25=0$ factor out $\Rightarrow$ $x^2(x-1)-25(x-1)=0 \Rightarrow (x-1)(x^2-25)=0$

$\left\langle \begin{array}{l} x-1=0 \Rightarrow x=1 \\ x^2-25=0 \Rightarrow x=\pm 5 \end{array}\right]$ three integer solutions

(c) $x^5-4x^3-3=0 \Rightarrow x^3(x^2-4)=3$ since x is integer x^3 and x^2-4 will both be integers

the product of two integers can be 3 only if those integers are ± 1 or ± 3

but x^3 can't be 3 since 3 is not cube of any integer number, so x can only be ± 1

$x=1 \longrightarrow 1^5-4\cdot 1^3-3 = 1-4-3=-6 \Rightarrow$ not solution

$x=-1 \longrightarrow (-1)^5-4\cdot(-1)^3-3 = -1+4-3=0 \Rightarrow -1$ is the only integer root.

3. (d) $(3x^8+x^2+1):(x^3-2x+1)$

$$
\begin{array}{r|l}
x^3-2x+1 & 3x^5+6x^3-3x^2+12x-12 \\ \hline
 & 3x^8 \qquad\qquad + x^2 \qquad + 1 \\
 & 3x^8 \quad -6x^6+3x^5 \\ \hline
 & // \quad 6x^6-3x^5 \quad +x^2 \quad +1 \\
 & \quad 6x^6 \quad -12x^4+6x^3 \\ \hline
 & // \; -3x^5+12x^4-6x^3+x^2 \quad +1 \\
 & \quad -3x^5 \quad +6x^3-3x^2 \\ \hline
 & // \quad 12x^4-12x^3+4x^2 \quad +1 \\
 & \quad 12x^4 \quad -24x^2+12x \\ \hline
 & // \quad -12x^3+28x^2-12x+1 \\
 & \quad -12x^3 \quad +24x+1 \\ \hline
 & // \quad 28x^2-36x
\end{array}
$$

$\dfrac{3x^8}{x^3}=3x^5$ $\quad \dfrac{6x^6}{x^3}=6x^3$

$\dfrac{-3x^5}{x^3}=-3x^2$ $\quad \dfrac{12x^4}{x^3}=12x$

$\dfrac{-12x^3}{x^3}=-12$

quotient $= 3x^5+6x^3-3x^2+12x-12$

remainder $= 28x^2-36x$

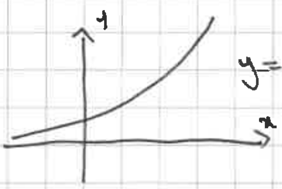
Section 4.9

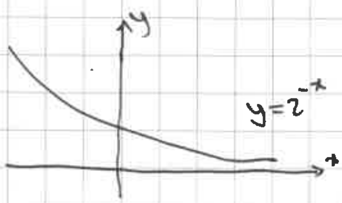
1. initial deposit = \$100 $\quad$ 12% interest $\quad$ after t years $\quad$ savings $= 100\left(1+\frac{12}{100}\right)^t$

t	1	2	5	10	20	30	50
savings	112	125	176	310	964	2996	28900

2.

x	-3	-2	-1	0	1	2	3
2^x	1/8	1/4	1/2	1	2	4	8
2^{-x}	8	4	2	1	1/2	1/4	1/8





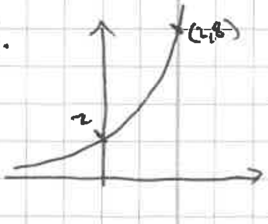
4. (a) $y=3^x$ exp $\quad$ (b) $y=x^{\sqrt{2}}$ not exp $\quad$ (c) $y=(\sqrt{2})^x$ exp

(d) $y=x^x$ not exp $\quad$ (e) $y=(2.7)^x$ exp $\quad$ (f) $y=\frac{1}{2^x}$ exp

5. inflation rate = 19% $\quad$ (a) $16\left(1+\frac{19}{100}\right)^5 = 38.18$ $\quad$ (b) $4.4\left(1+\frac{19}{100}\right)^{10} = 25.06$

(c) $250\,000\left(1+\frac{19}{100}\right)^4 = 501\,334.8$

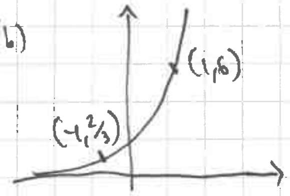
6.



$f(x)=C\,a^x$ we know from graph $f(0)=2$ $\quad f(2)=8$ so

$C\cdot a^0=2 \Rightarrow C=2$ $\qquad 2\cdot a^2=8 \Rightarrow a^2=4 \Rightarrow a=2$

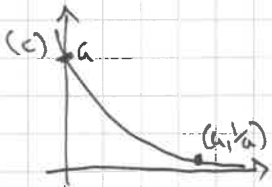
so $f(x)=2\cdot 2^x = 2^{x+1}$

(b) 

$f(x) = C\cdot a^x \qquad f(-1) = \frac{2}{3} \qquad f(1) = 6$

$$\begin{cases} C\cdot a^{-1} = \frac{2}{3} \\ C\cdot a^{1} = 6 \end{cases} \Rightarrow \begin{cases} \frac{C}{a} = \frac{2}{3} \\ Ca = 6 \end{cases} \Rightarrow \begin{cases} \frac{C^2}{6} = \frac{2}{3} \\ a = 6/C \end{cases} \Rightarrow \begin{cases} C^2 = 4 \\ a = 6/C \end{cases} \Rightarrow \begin{cases} C = 2 \\ a = 6/2 = 3 \end{cases}$$

$f(x) = 2\cdot 3^x$

(c) 

$f(0) = 4 \qquad f(x) = C\cdot a^x \Rightarrow \quad C\cdot a^0 = 4 \Rightarrow C = 4$

$f(4) = 1/4 \qquad 4\cdot a^4 = \frac{1}{4} \Rightarrow a^4 = \frac{1}{16} \Rightarrow a = \frac{1}{2} \Rightarrow f(x) = 4\cdot\frac{1}{2^x} = 2^{2-x}$

Section 4.10

1. (a) $\ln 9 = \ln 3^2 = 2\cdot\ln 3$ (b) $\ln\sqrt{3} = \ln 3^{1/2} = \frac{1}{2}\ln 3$

(c) $\ln\sqrt[5]{3^2} = \ln 3^{2/5} = \frac{2}{5}\ln 3$ (d) $\ln\left(\frac{1}{81}\right) = \ln 3^{-4} = -4\ln 3$

2. (a) $3^x = 8 \Rightarrow \ln 3^x = \ln 8 \Rightarrow x\cdot\ln 3 = \ln 8 \Rightarrow x = \frac{3\ln 2}{\ln 3}$

(b) $\ln x = 3 \Rightarrow e^{\ln x} = e^3 \Rightarrow x = e^3$

(c) $\ln(x^2-4x+5) = 0 \Rightarrow e^{\ln(x^2-4x+5)} = e^0 \Rightarrow x^2-4x+5 = 1 \Rightarrow x^2-4x+4=0 \Rightarrow x=2$

(d) $\ln(x(x-2)) = 0 \Rightarrow \ln(x(x-2)) = \ln 1 \Rightarrow x^2-2x = 1 \Rightarrow x^2-2x-1=0 \qquad x = 1\pm\sqrt{4+1} = 1\pm\sqrt{5}$

(e) $\frac{x\ln(x+3)}{x^2+1} = 0 \Rightarrow x\ln(x+3) = 0 \;\Big\langle\; \begin{matrix} x=0 \\ \ln(x+3)=0 \Rightarrow \ln(x+3) = \ln 1 \Rightarrow x+3=1 \Rightarrow x=-2 \end{matrix}$

(f) $\ln(\sqrt{x}-5) = 0 \Rightarrow \ln(\sqrt{x}-5) = \ln 1 \Rightarrow \sqrt{x}-5 = 1 \Rightarrow \sqrt{x} = 6 \Rightarrow x = 36$

5. $GDP_{CH} = 1.2\cdot 10^{12} \qquad GDP_{USA} = 5.6\cdot 10^{12} \qquad r = 0.09 \qquad s = 0.02 \qquad GDP_{CH} = Ae^{rt} \qquad GDP_{USA} = Be^{st}$

when are they equal? $Ae^{rt} = Be^{st} \Rightarrow \frac{e^{rt}}{e^{st}} = \frac{B}{A} \Rightarrow e^{(r-s)t} = \frac{B}{A} \Rightarrow (r-s)t = \log\frac{B}{A}$

$t = \frac{1}{r-s}\log\frac{B}{A} = \frac{1}{0.09-0.02}\log\frac{5.6}{1.2} = 22$ In 2012 they will be equal.

7. (a) $\exp(\ln x) - \ln(\exp(x)) = e^{\ln x} - \ln e^x = x - x\cdot\ln e = x - x = 0$

(b) $\ln(x^4\exp(-x)) = \ln(x^4 e^{-x}) = \ln x^4 + \ln e^{-x} = 4\ln x - x\ln e = 4\ln x - x$

(c) $\exp(\ln x^2 - 2\ln y) = e^{\ln x^2 - 2\ln y} = \frac{e^{\ln x^2}}{e^{2\ln y}} = \frac{x^2}{y^2}$