

Section 6.1

- From the figure, $f(3)=2$. The tangent line passes through $(3,2)$ and $(0,3)$ so it has slope $a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{0-3} = -\frac{1}{3}$. Then $f'(3) = -\frac{1}{3}$.
- $g(5)=1$. Tangent line goes through $(4,0)$ and $(5,1)$ so it has slope $a = \frac{1-0}{5-4} = 1$. Then $g'(5)=1$.

Section 6.2

$$1. f(x) = 4x^2 \quad \frac{f(s+h) - f(s)}{h} = \frac{4(s+h)^2 - 4s^2}{h} = \frac{4 \cdot (2s + 10h + h^2) - 100}{h} = \frac{100 + 40h + 4h^2 - 100}{h} = 40 + 4h$$

$$f'(s) = \lim_{h \rightarrow 0} (40 + 4h) = 40. \quad \text{We have } f'(x) = 8x \Rightarrow f'(5) = 8 \cdot 5 = 40.$$

$$4. C(x) = p + qx^2 \Rightarrow C'(x) = 2qx$$

$$5. f(x) = \frac{1}{x} \quad \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{1}{h} \cdot \frac{x - x - h}{x(x+h)} = -\frac{h}{hx(x+h)} = -\frac{1}{x(x+h)}$$

$$f'(x) = \lim_{h \rightarrow 0} \left(-\frac{1}{x(x+h)} \right) = -\frac{1}{x^2}$$

Section 6.3

$$1. (a) \lim_{x \rightarrow 0} (3 + 2x^2) = 3 + 2 \cdot 0^2 = 3$$

$$(b) \lim_{x \rightarrow -1} \frac{3 + 2x}{x - 1} = \frac{3 + 2 \cdot (-1)}{-1 - 1} = \frac{3 - 2}{-2} = -\frac{1}{2}$$

$$(c) \lim_{x \rightarrow 2} (2x^2 + 5)^3 = (2 \cdot 2^2 + 5)^3 = 13^3$$

$$4. (a) \lim_{x \rightarrow 2} (x^2 + 3x - 5) = 2^2 + 3 \cdot 2 - 5 = 4 + 6 - 5 = 5$$

$$(b) \lim_{y \rightarrow -3} \frac{1}{y + 8} = \frac{1}{-3 + 8} = \frac{1}{5}$$

$$(c) \lim_{x \rightarrow 0} \frac{x^3 - 2x - 1}{x^5 - x^2 - 1} = \frac{-1}{-1} = 1$$

$$5. (a) \lim_{h \rightarrow 2} \frac{\frac{1}{3} - \frac{2}{3h}}{h - 2} = \lim_{h \rightarrow 2} \frac{h - 2}{3h} \cdot \frac{1}{h - 2} = \lim_{h \rightarrow 2} \frac{1}{3h} = \frac{1}{6}$$

$$(b) \lim_{x \rightarrow 0} \frac{x^2 - 1}{x^2} = \frac{-1}{0} = -\infty$$

$$(c) \lim_{t \rightarrow 3} \frac{\sqrt[3]{32t - 96}}{t^2 - 2t - 3} = \lim_{t \rightarrow 3} \frac{\sqrt[3]{32} \cdot \sqrt[3]{t - 3}}{(t - 3)(t + 1)} = \lim_{t \rightarrow 3} \frac{\sqrt[3]{32}}{t + 1} \cdot \frac{1}{(t - 3)^{2/3}} = \frac{\sqrt[3]{32}}{4} \cdot \frac{1}{0} = +\infty$$

$$6. f(x) = x^2 + 2x \quad (a) \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{x^2 + 2x - 1 - 2}{x - 1} = \lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(x + 3)}{x - 1} = 4$$

$$(b) \lim_{x \rightarrow 2} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 2} \frac{x^2 + 2x - 3}{x - 1} = \frac{4 + 4 - 3}{2 - 1} = 5$$

Section 6.6

$$1. (a) y = 5 \Rightarrow y' = 0 \quad (b) y = x^4 \Rightarrow y' = 4x^3 \quad (c) y = 9x^{10} \Rightarrow y' = 9 \cdot 10x^9 = 90x^9 \quad (d) y = \pi^7 \Rightarrow y' = 0$$

$$2. (a) D(2g(x) + 3) = 2g'(x) \quad (b) D(-\frac{1}{6}g(x) + 8) = -\frac{1}{6}g'(x) \quad (c) D(\frac{g(x) - 5}{3}) = \frac{1}{3}g'(x)$$

$$3. (a) D(x^6) = 6x^5 \quad (b) D(3x^{11}) = 3 \cdot 11x^{10} = 33x^{10} \quad (c) D(x^{50}) = 50x^{49}$$

$$(d) D(-4x^{-7}) = -4 \cdot (-7)x^{-8} = 28x^{-8}$$

$$4. (a) \frac{d}{dr} (4\pi r^2) = 4\pi \cdot 2r = 8\pi r \quad (b) \frac{d}{dy} (Ay^{b+1}) = A \cdot b y^b \quad (c) \frac{d}{dA} \left(\frac{1}{A^2 \sqrt{A}} \right) = \frac{d}{dA} (A^{-2} \cdot A^{-\frac{1}{2}}) = \frac{d}{dA} (A^{-\frac{5}{2}}) = -\frac{5}{2} A^{-\frac{5}{2}-1} = -\frac{5}{2} A^{-\frac{7}{2}} = -\frac{5}{2A^{\frac{7}{2}} \sqrt{A}}$$