Section 6.7

2. (a) $D\left(\frac{3}{5}x^2 - 2x^7 + \frac{1}{8} - \sqrt{x}\right) = \frac{6}{5}x - 2\cdot 7x^6 - \frac{1}{2}x^{-\frac{1}{2}} = \frac{6}{5}x - 14x^6 - \frac{1}{2\sqrt{x}}$

(b) $D\left((2x^2-1)(x^4-1)\right) = 4x(x^4-1) + (2x^2-1)\cdot 4x^3 = 4x^5 - 4x + 8x^5 - 4x^3 = 12x^5 - 4x^3 - 4x$

(c) $D\left(\left(x^5+\frac{1}{x}\right)(x^5+1)\right) = \left(5x^4 - \frac{1}{x^2}\right)(x^5+1) + \left(x^5+\frac{1}{x}\right)\cdot 5x^4 = 5x^9 - x^3 + 5x^4 - \frac{1}{x^2} + 5x^9 + 5x^3$

3. (a) $D\left(\frac{1}{x^6}\right) = D(x^{-6}) = -6x^{-7}$ (b) $D\left(x^{-1}(x^2+1)\sqrt{x}\right) = D\left(x^{-\frac{1}{2}}(x^2+1)\right) =$

$= -\frac{1}{2}x^{-\frac{3}{2}}(x^2+1) + x^{-\frac{1}{2}}\cdot 2x = -\frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} + 2x^{\frac{1}{2}} = -\frac{1}{2\sqrt{x^3}} + \frac{3}{2}\sqrt{x}$

(c) $D\left(\frac{1}{\sqrt{x}^3}\right) = D\left(x^{3/2}\right) = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

(d) $D\left(\frac{x+1}{x-1}\right) = \frac{x-1-(x+1)}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2} = -\frac{2}{(x-1)^2}$

7. (a) $y = 3 - x - x^2$ at $x=1$ $\quad y = y(1) + y'(1)(x-1) \quad y(1) = 3-1-1 = 1$

$y'(x) = -1-2x \Rightarrow y'(1) = -1-2 = -3 \quad y = 1 - 3(x-1) = 1-3x+3 \Rightarrow 3x+y-4=0$

(b) $y = \frac{x^2-1}{x^2+1}$ at $x=1$ $\quad y(1) = \frac{1-1}{1+1} = 0 \quad y'(x) = \frac{2x(x^2+1)-(x^2-1)2x}{(x^2+1)^2} =$

$= \frac{\cancel{2x^3}+2x-\cancel{2x^3}+2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2} \quad y'(1) = \frac{4}{(1+1)^2} = \frac{4}{4} = 1 \quad y = 0 + 1\cdot(x-1) \Rightarrow x-y-1=$

Section 6.8

2. (a) $Y = -3(V+1)^5 \quad V = \frac{1}{3}t^3 \quad \frac{dY}{dt} = \frac{dY}{dV}\cdot\frac{dV}{dt} = -3\cdot 5(V+1)^4\cdot\frac{1}{\cancel{3}}\cdot\cancel{3}t^2 = -15(V+1)^4t^2 =$

$= -15\left(\frac{1}{3}t^3+1\right)^4 t^2$ (b) $K = AL^a \quad L = bt+c$

$\frac{\partial K}{\partial t} = \frac{\partial K}{\partial L}\cdot\frac{\partial L}{\partial t} = A\cdot aL^{a-1}\cdot b = A\cdot ab(bt+c)^{a-1}$

3. (a) $y = \frac{1}{(x^2+x+1)^5} = (x^2+x+1)^{-5} \quad z = x^2+x+1 \Rightarrow y' = D(z^{-5}) =$

$= -5z^{-6}\cdot z' = -5(x^2+x+1)^{-6}\cdot(2x+1) = -\frac{5(2x+1)}{(x^2+x+1)^6}$

17. (a) $D(x+f(x)) = 1+f'(x)$ (b) $D\left((f(x))^2 - x\right) = 2f(x)\cdot f'(x) - 1$

(c) $D\left(f(x)^4\right) = 4f(x)^3\cdot f'(x)$ (d) $D\left(x^2 f(x) + f(x)^3\right) =$

$= 2x f(x) + x^2 f'(x) + 3f(x)^2\cdot f'(x)$

Section 6.9

4. $g(t) = \frac{t^2}{t-1} \quad g'(t) = \frac{2t(t-1)-t^2\cdot 1}{(t-1)^2} = \frac{2t^2-2t-t^2}{(t-1)^2} = \frac{t^2-2t}{(t-1)^2} = \frac{t^2-2t}{t^2-2t+1}$

$g''(t) = \frac{(2t-2)(t^2-2t+1)-(t^2-2t)(2t-2)}{(t-1)^4} = \frac{2\cancel{(t-1)}(\cancel{t^2}-\cancel{2t}+1-\cancel{t^2}+\cancel{2t})}{(t-1)^{\cancel{4}3}} = \frac{2}{(t-1)^3} \Rightarrow g''(2)=2$

Section 6.10

1. (a) $y = e^x + x^2 \Rightarrow y' = e^x + 2x$ (b) $y = 5e^x - 3x^3 + 8 \Rightarrow y' = 5e^x - 3\cdot 3x^2 = 5e^x - 9x^2$

(c) $y = \frac{x}{e^x} = x\cdot e^{-x} \Rightarrow y' = 1\cdot e^{-x} + x\cdot(-e^{-x}) = e^{-x} - xe^{-x} = \frac{1-x}{e^x}$

(d) $y = \frac{x+x^2}{e^x+1} \Rightarrow y' = \frac{(1+2x)(e^x+1)-(x+x^2)\cdot e^x}{(e^x+1)^2} = \frac{e^x+2xe^x+1+2x-xe^x-x^2e^x}{(e^x+1)^2} = \frac{-x^2e^x+xe^x+e^x+2x+1}{(e^x+1)^2}$

6. (a) $\frac{d}{dx}(e^{e^x})$ we take $z = e^x \Rightarrow \frac{d}{dx}(e^z) = \frac{d}{dz}(e^z)\cdot z' = e^z \cdot e^x = e^{e^x}\cdot e^x$

(b) $\frac{d}{dt}(e^{t/2}+e^{-t/2}) = \frac{1}{2}e^{t/2} - \frac{1}{2}e^{-t/2} = \frac{e^{t/2}-e^{-t/2}}{2}$

(c) $\frac{d}{dt}\left(\frac{1}{e^t+e^{-t}}\right) = \frac{-(e^t-e^{-t})}{(e^t+e^{-t})^2} = \frac{e^{-t}-e^t}{(e^t+e^{-t})^2}$

(d) $\frac{d}{dz}((e^{z^3}-1)^{\frac{1}{3}}) = \frac{1}{\cancel{3}}(e^{z^3}-1)^{\frac{1}{3}-1}\cdot e^{z^3}\cdot \cancel{3}z^2 = (e^{z^3}-1)^{-\frac{2}{3}}e^{z^3}z^2$

S6.11 2. (a) $y = x^3(\ln x)^2 \Rightarrow y' = 3x^2(\ln x)^2 + x^{\cancel{3}2}\cdot 2\ln x\cdot\frac{1}{\cancel{x}} = 3x^2(\ln x)^2 + 2x^2\ln x$

(b) $y = \frac{x^2}{\ln x}$ $\quad y' = \frac{2x\ln x - x^2\cdot\frac{1}{x}}{(\ln x)^2} = \frac{2x\ln x - x}{(\ln x)^2}$

(c) $y = (\ln x)^{10}$ $\quad y' = 10(\ln x)^9\cdot\frac{1}{x}$

(d) $y = (\ln x + 3x)^2 \Rightarrow y' = 2(\ln x + 3x)(\frac{1}{x}+3)$

Section 7.1

1. $3x^2+2y=5$ derive both sides $D(3x^2+2y) = D(5)$

$6x + 2y' = 0 \Rightarrow 2y' = -6x \Rightarrow y' = -3x$

$3x^2+2y=5 \Rightarrow y = \frac{5-3x^2}{2} \Rightarrow y' = -\frac{3}{2}\cdot 2x = -3x$

2. $x^2y = 1$ derive both sides $\Rightarrow D(x^2y) = D(1) \Rightarrow 2xy + x^2y' = 0$

$x^2y' = -2xy \Rightarrow y' = -2\frac{y}{x} \Rightarrow y'' = -2\frac{y'\cdot x - y}{x^2} = \frac{2(y-xy')}{x^2}$

8. (a) $xy = g(x)+y^3 \Rightarrow D(xy) = D(g(x)+y^3) \Rightarrow xy'+y = g'(x) + 3y^2\cdot y'$

$\Rightarrow xy' - 3y^2y' = g'(x) - y \Rightarrow (x-3y^2)y' = g'(x)-y \Rightarrow y' = \frac{g'(x)-y}{x-3y^2}$

(b) $g(x+y) = x^2+y^2 \Rightarrow D(g(x+y)) = D(x^2+y^2) \Rightarrow g'(x+y)\cdot(1+y') = 2x+2yy'$

$g'(x+y) + g'(x+y)\cdot y' = 2x + 2y\cdot y' \Rightarrow g'(x+y)y' - 2y\cdot y' = 2x - g'(x+y)$

$(g'(x+y)-2y)y' = 2x - g'(x+y) \Rightarrow y' = \frac{2x-g'(x+y)}{g'(x+y)-2y}$

(c) $(xy+1)^2 = g(x^2y) \Rightarrow D((xy+1)^2) = D(g(x^2y)) \Rightarrow$

$\Rightarrow 2(xy+1)(xy'+y) = g'(x^2y)\cdot(2xy + x^2y')$

$2x(xy+1)y' + 2(xy+1)y = g'(x^2y)2xy + g'(x^2y)x^2y'$

$(2x(xy+1) - x^2)y' = g'(x^2y)2xy - 2(xy+1)y$

$y' = \frac{g'(x^2y)2xy - 2y(xy+1)}{2x(xy+1)-x^2}$