Section 7.4

2. $f(x) = (5x+3)^{-2}$ $\qquad f(x) \approx f(0) + f'(0)x \qquad f(0) = 3^{-2} = \frac{1}{9}$

$f'(x) = -2(5x+3)^{-3} \cdot 5 = -10(5x+3)^{-3} \Rightarrow f'(0) = -10 \cdot 3^{-3} = -\frac{10}{27}$

$f(x) \approx \frac{1}{9} - \frac{10}{27}x$

3. (a) $f(x) = (1+x)^{-1} \Rightarrow f(0) = 1 \qquad f'(x) = -(1+x)^{-2} \Rightarrow f'(0) = -1 \qquad f(x) \approx 1 - x$

(b) $f(x) = (1+x)^5 \Rightarrow f(0) = 1 \qquad f'(x) = 5(1+x)^4 \Rightarrow f'(0) = 5 \qquad f(x) \approx 1 + 5x$

(c) $f(x) = (1-x)^{1/4} \Rightarrow f(0) = 1 \qquad f'(x) = \frac{1}{4}(1-x)^{-3/4} \cdot (-1) = -\frac{1}{4}(1-x)^{-3/4} \Rightarrow f'(0) = -\frac{1}{4}$

$f(x) \approx 1 - \frac{1}{4}x$

6. (a) $f(x) = (1+x)^m \qquad f(x) \approx f(0) + f'(0)x$ for x close to $0 \qquad f(0) = 1$

$f'(x) = m(1+x)^{m-1} \Rightarrow f'(0) = m \qquad f(x) \approx 1 + mx$

(b) (i) $\sqrt[3]{1.1} = \left(1 + \frac{1}{10}\right)^{1/3} \approx 1 + \frac{1}{3} \cdot \frac{1}{10} = 1 + \frac{1}{30}$ $\qquad$ (ii) $\sqrt[5]{33} = 2\left(1 + \frac{1}{32}\right)^{1/5} \approx 2\left(1 + \frac{1}{5} \cdot \frac{1}{32}\right)$

(iii) $\sqrt[3]{9} = \sqrt[3]{1+8} = \sqrt[3]{8\left(1+\frac{1}{8}\right)} = 2\left(1 + \frac{1}{8}\right)^{1/3} \approx 2\left(1 + \frac{1}{3} \cdot \frac{1}{8}\right) = 2\left(1 + \frac{1}{24}\right) = 2 + \frac{1}{12}$

(iv) $(0.98)^{25} = (1 - 0.02)^{25} = \left(1 - \frac{1}{50}\right)^{25} \approx 1 - \frac{1}{50} \cdot 25 = 1 - \frac{1}{2} = \frac{1}{2}$

8. $3xe^{xy^2} - 2y = 3x^2 + y^2 \qquad (x,y) = (1,0)$

(a) $D(3xe^{xy^2} - 2y) = D(3x^2 + y^2) \Rightarrow 3e^{xy^2} + 3x\, e^{xy^2}(y^2 + x \cdot 2y \cdot y') - 2y' =$

$= 6x + 2y \cdot y' \Rightarrow 3e^{xy^2} + 3xy^2 e^{xy^2} + 6x^2 y e^{xy^2} \cdot y' - 2y' = 6x + 2y \cdot y'$

$(6x^2 y e^{xy^2} - 2 - 2y)y' = -3e^{xy^2} - 3xy^2 e^{xy^2} + 6x$ $\qquad$ substitute $x = 1,\ y = 0$

$-2y' = -3 + 6 \Rightarrow y' = \frac{3}{-2} = -\frac{3}{2}$

(b) $y \approx -\frac{3}{2}(x-1)$ around $x = 1$

Section 7.5

1. (a) $f(x) = (1+x)^5 \qquad x = 0 \qquad f(0) = 1 \qquad f'(x) = 5(1+x)^4 \qquad f'(0) = 5 \qquad f''(x) = 20(1+x)^3$

$f''(0) = 20 \qquad f(x) \approx 1 + 5x + 10x^2$

(b) $F(K) = AK^{\alpha}$ about $K = 1 \qquad F(1) = A \qquad F'(K) = A\alpha K^{\alpha-1} \qquad F'(1) = A\alpha$

$F''(K) = A\alpha(\alpha-1)K^{\alpha-2} \qquad F''(1) = A\alpha(\alpha-1) \qquad F(K) \approx A + A\alpha(K-1) + \frac{1}{2}A\alpha(\alpha-1)(K-1)^2$

(c) $f(\varepsilon) = \left(1 + \frac{3}{2}\varepsilon + \frac{1}{2}\varepsilon^2\right)^{1/2} \qquad \varepsilon = 0 \qquad f(0) = 1 \qquad f'(\varepsilon) = \frac{1}{2}\left(1 + \frac{3}{2}\varepsilon + \frac{1}{2}\varepsilon^2\right)^{-\frac{1}{2}}\left(\frac{3}{2} + \varepsilon\right)$

$f'(0) = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4} \qquad f''(\varepsilon) = \frac{1}{2} \cdot \left(-\frac{1}{2}\right)\left(1 + \frac{3}{2}\varepsilon + \frac{1}{2}\varepsilon^2\right)^{-\frac{3}{2}}\left(\frac{3}{2} + \varepsilon\right)^2 + \frac{1}{2}\left(1 + \frac{3}{2}\varepsilon + \frac{1}{2}\varepsilon^2\right)^{-\frac{1}{2}}$

$f''(0) = -\frac{1}{4} \cdot \frac{9}{4} + \frac{1}{2} = -\frac{9}{16} + \frac{1}{2} = -\frac{1}{16} \Rightarrow f(\varepsilon) \approx 1 + \frac{3}{4}\varepsilon - \frac{1}{32}\varepsilon^2$

(d) $H(x) = (1-x)^{-1} \qquad x = 0 \qquad H(0) = 1 \qquad H'(x) = -(1-x)^{-2}(-1) = (1-x)^{-2} \qquad H'(0) = 1$

$H''(x) = -2(1-x)^{-3}(-1) = 2(1-x)^{-3} \qquad H''(0) = 2 \qquad H(x) \approx 1 + x + x^2$

2. $f(x)=\ln(1+x)$ about $x=0$ $\quad f(x)\approx f(0)+f'(0)x+\frac{f''(0)}{2}x^2+\frac{f'''(0)}{6}x^3+\frac{f^{iv}(0)}{24}x^4+\frac{f^{v}(0)}{120}x^5$

$f(0)=\ln 1=0 \quad f'(x)=\frac{1}{1+x} \quad f'(0)=1 \quad f''(x)=-\frac{1}{(1+x)^2} \quad f''(0)=-1$

$f'''(x)=-(-2)(1+x)^{-3}=2(1+x)^{-3} \quad f'''(0)=2 \quad f^{iv}(x)=2\cdot(-3)(1+x)^{-4}=-6(1+x)^{-4}$

$f^{iv}(0)=-6 \quad f^{v}(x)=-6\cdot(-4)(1+x)^{-5}=24(1+x)^{-5} \quad f^{v}(0)=24$

$f(x)\approx x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\frac{1}{5}x^5$

3. $f(x)=5\left(\ln(1+x)-\sqrt{1+x}\right)$ about $x=0$ $\quad f(x)\approx f(0)+f'(0)x+\frac{1}{2}f''(0)x^2$

$f(0)=5(\ln 1-1)=-5 \quad f'(x)=5\left(\frac{1}{1+x}-\frac{1}{2\sqrt{1+x}}\right) \quad f'(0)=5\left(1-\frac{1}{2}\right)=\frac{5}{2}$

$f''(x)=5\left(-\frac{1}{(1+x)^2}+\frac{1}{4\sqrt{(1+x)^3}}\right) \quad f''(0)=5\left(-1+\frac{1}{4}\right)=5\cdot\left(-\frac{3}{4}\right)=-\frac{15}{4}$

$f(x)\approx -5+\frac{5}{2}x-\frac{15}{8}x^2$

5. $1+x^3y+x=y^{1/2}$ about $(x,y)=(0,1)$ $\quad x=0,\ y=1\rightarrow 1+0+0=1^{1/2}$ ✓

so $y(0)=1$ $\quad D(1+x^3y+x)=D(y^{1/2}) \Rightarrow 3x^2y+x^3y'+1=\frac{1}{2}y^{-1/2}\cdot y'$

$x=0,\ y=1\rightarrow 1=\frac{1}{2}y' \Rightarrow y'(0)=2$ $\quad$ we derive this once more

$6xy+3x^2y'+3x^2y'+x^3y''=-\frac{1}{4}y^{-3/2}\cdot(y')^2+\frac{1}{2}y^{-\frac{1}{2}}\cdot y''$

$x=0,\ y=1,\ y'=2 \rightarrow 0=-\frac{1}{4}\cdot 1\cdot 4+\frac{1}{2}\cdot 1\cdot y'' \Rightarrow \frac{1}{2}y''-1=0 \Rightarrow y''(0)=2$

$y(x)\approx 1+2x+x^2$

Section 7.6

1. $n=2$ $\quad f(x)=\ln(1+x)$ $\quad f(x)\approx f(0)+f'(0)x+\frac{f''(0)}{2}x^2$

$f(0)=\ln 1=0 \quad f'(x)=\frac{1}{1+x} \quad f'(0)=1 \quad f''(x)=-\frac{1}{(1+x)^2} \quad f''(0)=-1$

$f(x)\approx x-\frac{1}{2}x^2$

2. $(1+x)^m\approx 1+mx+\frac{1}{2}m(m-1)x^2$ $\quad \sqrt[3]{25}=3\left(1-\frac{2}{27}\right)^{1/3}$ $\quad$ put $m=\frac{1}{3},\ x=-\frac{2}{27}$

$\sqrt[3]{25}\approx 3\left(1-\frac{1}{3}\cdot\frac{2}{27}+\frac{1}{2}\cdot\frac{1}{3}\left(\frac{1}{3}-1\right)\cdot\frac{4}{27^2}\right)=3-\frac{2}{27}-\frac{4}{3\cdot 27^2}$

$\sqrt[5]{33}=2\left(1+\frac{1}{32}\right)^{1/5}\approx 2\left(1+\frac{1}{5}\cdot\frac{1}{32}+\frac{1}{2}\cdot\frac{1}{5}\cdot\left(-\frac{4}{5}\right)\cdot\frac{1}{32^2}\right)$