Section 7.1

5. $2x^2+6xy+y^2=18$. Find y' and y'' at $(x,y)=(1,2)$. We derivate both sides

$D(2x^2+6xy+y^2)=D(18) \Rightarrow 4x+6y+6xy'+2y\cdot y'=0$ $\quad x=1, y=2 \to 4\cdot1+6\cdot2+6\cdot y'+2\cdot2\cdot y'=0$

$4+12+6y'+4y'=0 \Rightarrow 10y'=-16 \Rightarrow y'=-8/5$. Now we derivate again

$D(4x+6y+6xy'+2y\,y')=0 \Rightarrow 4+6y'+6y'+6xy''+2(y')^2+2y\cdot y''=0$

$x=1, y=2, y'=-8/5 \Rightarrow 4-\frac{48}{5}-\frac{48}{5}+6y''+2\cdot\frac{64}{25}+2\cdot2\,y''=0$

$10y''=-4+\frac{96}{5}-\frac{128}{25}=\frac{-100+480-128}{25}=\frac{252}{25} \Rightarrow y''=\frac{252}{250}=\frac{126}{125}$

7. $2xy-3y^2=9$ $\quad$ (a) $(x,y)=(6,1)$ $\quad D(2xy-3y^2)=D(9) \Rightarrow 2y+2xy'-6y\cdot y'=0$

$\cancel{2}(x-3y)y'=-\cancel{2}y \Rightarrow y'=-\frac{y}{x-3y} \Rightarrow x=6, y=1 \quad y'=-\frac{1}{6-3}=-\frac{1}{3}$

$D(y+xy'-3y\cdot y')=0 \Rightarrow y'+y'+xy''-3(y')^2-3y\cdot y''=0$ $\quad x=6, y=1, y'=-\frac{1}{3}$

$\to -\frac{2}{3}+6y''-3\cdot\frac{1}{9}-3y''=0 \Rightarrow 3y''=\frac{2}{3}+\frac{1}{3} \Rightarrow y''=\frac{1}{3}$

Section 7.8

2. $f(x)=\begin{cases} x^2-1 & \text{for } x\leq 0 \\ -x^2 & \text{for } x>0 \end{cases}$ $\quad \lim_{x\to0^-}f(x)=\lim_{x\to0^-}(x^2-1)=-1$ $\quad \lim_{x\to0^+}f(x)=\lim_{x\to0^+}(-x^2)=0$

Since the two limits are different, $f(x)$ is <u>not</u> continuous at $x=0$

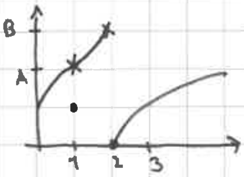
$g(x)=\begin{cases} 3x-2 & \text{for } x\leq 2 \\ -x+6 & \text{for } x>2 \end{cases}$ $\quad \lim_{x\to2^-}g(x)=\lim_{x\to2^-}(3x-2)=6-2=4$ $\quad \lim_{x\to2^+}g(x)=\lim_{x\to2^+}(-x+6)=4$

Since the two limits have same value, $g(x)$ is continuous at $x=2$.

3. (a) every $x\in\mathbb{R}$ $\quad$ (b) $1-x\neq0 \Rightarrow x\neq1$ $\quad$ (c) $\sqrt{2-x}\neq0 \Rightarrow x\neq2$ (also $2-x\geq0 \Rightarrow x<2$)

(d) every $x\in\mathbb{R}$ $\quad$ (e) $x^2+2x-2\neq0 \Rightarrow x\neq-1\pm\sqrt{1+2} \Rightarrow x\neq-1\pm\sqrt{3}$ $\quad$ (f) $\begin{cases} x>0 \\ x+2>0 \end{cases} \Rightarrow x>0$

Section 7.9

1. 

(a) $\lim_{x\to1^-}f(x)=A$ $\quad$ (b) $\lim_{x\to1^+}f(x)=A$

(c) $\lim_{x\to2^-}f(x)=B$ $\quad$ (d) $\lim_{x\to2^+}f(x)=0$

2. (a) $\lim_{x\to0^+}(x^2+3x-4)=-4$ $\quad$ (b) $\lim_{x\to0^-}\frac{x+|x|}{x}=\lim_{x\to0^-}\left(1+\frac{|x|}{x}\right)=1-1=0$

(c) $\lim_{x\to0^+}\frac{x+|x|}{x}=\lim_{x\to0^+}\left(1+\frac{|x|}{x}\right)=2$ $\quad$ (d) $\lim_{x\to0^+}\frac{-1}{\sqrt{x}}=0$ $\quad$ (e) $\lim_{x\to3^+}\frac{x}{x-3}=+\infty$

(f) $\lim_{x\to3^-}\frac{x}{x-3}=-\infty$

Section 7.12

1. (a) $\lim_{x\to3}\frac{3x^2-27}{x-3}=\left[\frac{0}{0}\right]=\lim_{x\to3}\frac{6x}{1}=18$ $\quad$ (b) $\lim_{x\to0}\frac{e^x-1-x-\frac{1}{2}x^2}{3x^3}=\left[\frac{0}{0}\right]=$

$=\lim_{x\to0}\frac{e^x-1-x}{9x^2}=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{e^x-1}{18x}=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{e^x}{18}=\frac{1}{18}$

(c) $\lim_{x\to0}\frac{e^{-3x}-e^{-2x}+x}{x^2}=\lim_{x\to0}\frac{-3e^{-3x}+2e^{-2x}+1}{2x}=\lim_{x\to0}\frac{9e^{-3x}-4e^{-2x}}{2}=\frac{9-4}{2}=\frac{5}{2}$

2. (a) $\lim_{x\to a} \frac{x^2-a^2}{x-a} = \left[\frac{0}{0}\right] = \lim_{x\to a} \frac{2x}{1} = 2a$

(b) $\lim_{x\to 0} \frac{2\sqrt{1+x}-2-x}{2\sqrt{1+x+x^2}-2-x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\frac{1}{\sqrt{1+x}}-1}{\frac{2x+1}{\sqrt{1+x+x^2}}-1} = \left[\frac{0}{0}\right] =$

$= \lim_{x\to 0} \frac{-\frac{1}{2}(1+x)^{-3/2}}{-\frac{1}{2}(1+x+x^2)^{-3/2}(2x+1)^2 + \frac{2}{\sqrt{1+x+x^2}}} = \frac{-1/2}{-\frac{1}{2}+2} = -\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3} = -\frac{1}{3}$

3. (a) $\lim_{x\to 1} \frac{x-1}{x^2-1} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{1}{2x} = \frac{1}{2}$

(b) $\lim_{x\to -2} \frac{x^3+3x^2-4}{x^3+5x^2+8x+4} = \left[\frac{0}{0}\right] =$

$= \lim_{x\to -2} \frac{3x^2+6x}{3x^2+10x+8} = \left[\frac{0}{0}\right] = \lim_{x\to -2} \frac{6x+6}{6x+10} = \frac{6(-2)+6}{6(-2)+10} = \frac{-6}{-2} = 3$

(c) $\lim_{x\to 2} \frac{x^4-4x^3+6x^2-8x+8}{x^3-3x^2+4} = \left[\frac{0}{0}\right] = \lim_{x\to 2} \frac{4x^3-12x^2+12x-8}{3x^2-6x} = \left[\frac{0}{0}\right] =$

$= \lim_{x\to 2} \frac{12x^2-24x+12}{6x-6} = \frac{12\cdot 4-24\cdot 2+12}{6\cdot 2-6} = \frac{12}{6} = 2$

(d) $\lim_{x\to 1} \frac{\ln x - x+1}{(x-1)^2} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{1/x - 1}{2(x-1)} = \left[\frac{0}{0}\right] =$

$= \lim_{x\to 1} \frac{-1/x^2}{2} = -\frac{1}{2}$

(e) $\lim_{x\to 1} \frac{1}{x-1} \ln\left(\frac{7x+1}{4x+4}\right) = [\infty\cdot 0] = \lim_{x\to 1} \frac{\ln\left(\frac{7x+1}{4x+4}\right)}{x-1} =$

$= \lim_{x\to 1} \frac{\cancel{4x+4}}{7x+1}\cdot\frac{7(4x+4)-4(7x+1)}{(4x+4)^{\cancel{2}}} = \lim_{x\to 1} \frac{\cancel{28x}+28-\cancel{28x}-4}{(4x+4)(7x+1)} = \frac{\cancel{24}^{3}}{8\cdot\cancel{8}} = \frac{3}{8}$

(f) $\lim_{x\to 1} \frac{x^x - x}{1-x+\ln x} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{e^{x\ln x}-x}{1-x+\ln x} = \lim_{x\to 1} \frac{x^x(\ln x+1)-1}{-1+\frac{1}{x}} = \left[\frac{0}{0}\right] =$

$= \lim_{x\to 1} \frac{x^x(\ln x+1)^2 + x^x\cdot\frac{1}{x}}{-1/x^2} = \lim_{x\to 1} -x^2\cdot\left(x^x(\ln x+1)^2 + x^{x-1}\right) = -(1+1) = -2$