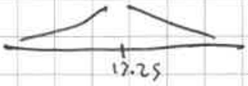
Section 9.2

1. $y = -2.05 + 1.06x - 0.04x^2$ $y' = 1.06 - 0.08x$ $y' = 0 \Leftrightarrow x = \frac{1.06}{0.08} = 13.25$

for $x=0$ $y' = 1.06 > 0$, for $x = 20$ $y' = 1.06 - 1.6 < 0$ so $y' > 0$ for $x < 13.25$

$y' < 0$ for $x > 13.25$

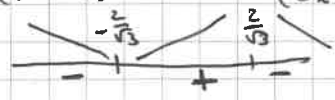


$x = 13.25$ is a max point.

2. $h(x) = \frac{8x}{3x^2+4}$ we can see that $\lim_{x\to\pm\infty} h(x) = 0$.

$h'(x) = \frac{8(3x^2+4) - 8x\cdot 6x}{(3x^2+4)^2} = \frac{8(3x^2+4-6x^2)}{(3x^2+4)^2} = \frac{8(4-3x^2)}{(3x^2+4)^2}$ $h'(x) = 0 \Leftrightarrow 4 - 3x^2 = 0$

$\Leftrightarrow x^2 = \frac{4}{3} \Leftrightarrow x = \pm\frac{2}{\sqrt3}$



$x = -2 \to 4 - 3\cdot 4 = -8 < 0$, $x = 0 \to 4 - 3\cdot 0 = 4 > 0$, $x = 2 \to 4 - 3\cdot 4 = -8 < 0$

$x = -\frac{2}{\sqrt3}$ is a min point, $x = \frac{2}{\sqrt3}$ is a max point.

5. $g(x) = x^3 \ln x$ for $x \in (0,\infty)$ $g'(x) = 3x^2 \ln x + x^3\cdot\frac{1}{x} = x^2(3\ln x + 1)$

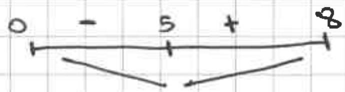
$g'(x) = 0 \Rightarrow$ either $x = 0$ or $3\ln x + 1 = 0 \Leftrightarrow \ln x = -\frac{1}{3} \Leftrightarrow x = e^{-1/3}$

Since $x \in (0,\infty)$, $x = 0$ is not acceptable, so $x = e^{-1/3}$ is the only possible extreme point.

Section 9.6

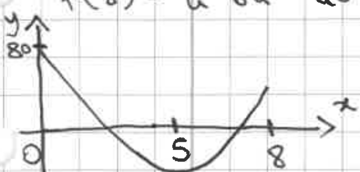
1. $f(x) = 4x^2 - 40x + 80$ for $x \in [0,8]$ $f'(x) = 8x - 40$ $f'(x) = 0 \Leftrightarrow x = \frac{40}{8} = 5$

$f'(0) = -40 < 0$ $f'(6) = 8\cdot 6 - 40 = 8 > 0$



$x = 5$ is min point max point is either 0 or 8. $f(0) = 80$

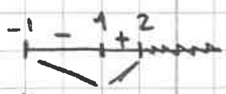
$f(8) = 4\cdot 64 - 40\cdot 8 + 80 = 256 - 320 + 80 = 336 - 320 = 16$ So $x = 0$ is max point.



$f(5) = 4\cdot 25 - 40\cdot 5 + 80 = 100 - 200 + 80 = -20$

2. (a) $f(x) = -2x - 1$ over $[0,3]$ $f'(x) = -2 < 0 \Rightarrow f$ decreasing $\Rightarrow x = 0$ max, $x = 3$ min

(b) $f(x) = x^3 - 3x + 8$ over $[-1,2]$ $f'(x) = 3x^2 - 3$ $f'(x) = 0$ for $x^2 = 1 \Leftrightarrow x = \pm 1$



$f'(0) = -3 < 0$ $f'(2) = 3\cdot 4 - 3 = 9 > 0$ $x = 1$ is min, max is either -1 or 2

$f(-1) = -1 + 3 + 8 = 10$ $f(2) = 8 - 6 + 8 = 10$ both $x = -1$ and $x = 2$ are max points.

(c) $f(x) = \frac{x^2+1}{x}$ over $[\frac{1}{2}, 2]$ $f(x) = x + \frac{1}{x} \Rightarrow f'(x) = 1 - \frac{1}{x^2}$ $f'(x) = 0 \Leftrightarrow 1 = \frac{1}{x^2} \Leftrightarrow x = \pm 1$

$\frac{1}{2}$ 1 2

$f'(\frac{1}{2}) = 1 - \frac{1}{1/4} = 1 - 4 = -3 < 0$ $f'(2) = 1 - \frac{1}{4} = \frac{3}{4} > 0$ $x = 1$ min

$f(\frac{1}{2}) = \frac{1}{2} + 2 = \frac{5}{2}$ $f(2) = 2 + \frac{1}{2} = \frac{5}{2}$ both $x = \frac{1}{2}$ and $x = 2$ are max

(d) $f(x) = x^5 - 5x^3$ over $[-1,\sqrt5]$ $f'(x) = 5x^4 - 15x^2$ $f'(x) = 0 \Leftrightarrow x = 0$ or $x^2 = 3 \Leftrightarrow x = \pm\sqrt3$

-1 0 $\sqrt3$ $\sqrt5$ $-$ $+$

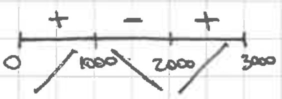
$f'(-1) = 5 - 15 = -10 < 0$ $f'(1) = 5 - 15 = -10 < 0$ $f(2) = 5\cdot 16 - 15\cdot 4 = 20(4-3) = 20 > 0$

$x=\sqrt{3}$ is min, max is either $x=-1$ or $x=\sqrt{5}$ $\quad f(-1)=-1+5=4$

$f(\sqrt{5})=25\sqrt{5}-5\cdot5\sqrt{5}=0$ so $x=-1$ is max point

(e) $f(x)=x^3-4500x^2+6\cdot10^6x$ over $[0,3000]$ $\quad f'(x)=3x^2-9000x+6\cdot10^6$

$$f'(x)=0 \iff x=\frac{9000\pm\sqrt{81\cdot10^6-72\cdot10^6}}{6}=\frac{9000\pm3000}{6}=\begin{cases}\frac{12000}{6}=2000\\ \frac{6000}{6}=1000\end{cases}$$



$f'(0)=6\cdot10^6>0$ $\quad f'(1500)=-750000<0$

$f'(3000)=27\cdot10^6-27\cdot10^6+6\cdot10^6=6\cdot10^6>0$

max is either $x=1000$ or $x=3000$ $\quad f(1000)=10^9-4{,}5\cdot10^9+6\cdot10^9=3{,}5\cdot10^9$

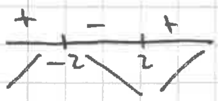
$f(3000)=27\cdot10^9-4{,}5\cdot9\cdot10^9+18\cdot10^9=4{,}5\cdot10^9$ $\quad x=3000$ is max point

min is either 0 or 2000 $\quad f(0)=0$ $\quad f(2000)=8\cdot10^9-4{,}5\cdot4\cdot10^9+12\cdot10^9=2\cdot10^9$

$x=0$ is min point.

Section 9.6

1. $f(x)=x^3-12x$ $\quad f'(x)=3x^2-12$ $\quad f'(x)=0 \iff x^2=4 \iff x=\pm2$



$f'(-3)=3\cdot9-12>0$ $\quad f'(0)=-12<0$ $\quad f'(3)=3\cdot9-12>0$

$x=-2$ max point, $x=2$ min point $\quad f''(x)=6x$

$f''(-2)=-12<0 \Rightarrow$ max $\quad f''(2)=12>0 \Rightarrow$ min.

2. (a) $f(x)=-2x-1 \Rightarrow f'(x)=-2$ $\quad f$ is always decreasing, so no possible extreme points

(b) $f(x)=x^3-3x+8$ $\quad f'(x)=3x^2-3$ $\quad f'(x)=0 \iff x^2=1 \iff x=\pm1$ possible extreme points

(c) $f(x)=x+\frac{1}{x} \Rightarrow f'(x)=1-\frac{1}{x^2}$ $\quad f'(x)=0 \iff 1=\frac{1}{x^2} \iff x^2=1 \iff x=\pm1$

(d) $f(x)=x^5-5x^3$ $\quad f'(x)=5x^4-15x^2$ $\quad f'(x)=0 \iff x=0$ or $x^2=3 \iff x=\pm\sqrt{3}$

(e) $f(x)=\frac{1}{2}x^2-3x+5$ $\quad f'(x)=x-3$ $\quad f'(x)=0 \iff x=3$

(f) $f(x)=x^3+3x^2-2$ $\quad f'(x)=3x^2+6x$ $\quad f'(x)=0 \iff x=0$ or $x=-2$