Section 10.1

1. (a) $\int x^{13}dx = \frac{x^{14}}{14}+c$ (b) $\int x\sqrt{x}\,dx = \int x^{3/2}dx = \frac{x^{\frac32+1}}{\frac32+1}+c = \frac{2x^{5/2}}{5}+c = \frac25 x^2\sqrt{x}+c$

(c) $\int \frac{1}{\sqrt x}dx = \int x^{-1/2}dx = \frac{x^{-\frac12+1}}{-\frac12+1}+c = 2x^{\frac12}+c = 2\sqrt x + c$

(e) $\int e^{-x}dx = -e^{-x}+c$ (f) $\int e^{x/4}dx = e^{x/4}\cdot\frac{1}{1/4}+c = 4e^{x/4}+c$ (g) $\int 3e^{-2x}dx = 3e^{-2x}\cdot\frac{1}{-2}+c = -\frac32 e^{-2x}+c$

2. (a) $C'(x)=3x+4$, $C(0)=40 \Rightarrow C(x)=\frac32x^2+4x+c$ $C(0)=40 \Leftrightarrow c=40$

$C(x)=\frac32x^2+4x+40$ (b) $C'(x)=ax+b$ $C(x)=\frac a2 x^2+bx+c_0$

3. (a) $\int(x^3+2x-3)dx = \frac{x^4}{4}+x^2-3x+c$ (b) $\int(x-1)^2dx = \frac{(x-1)^3}{3}+c$

(c) $\int(x-1)(x+2)dx = \int(x^2-x+2x-2)dx = \int(x^2+x-2)dx = \frac{x^3}{3}+\frac{x^2}{2}-2x+c$

(d) $\int(x+2)^3dx = \frac{(x+2)^4}{4}+c$ (e) $\int(e^{3x}-e^{2x}+e^x)dx = \frac13e^{3x}-\frac12e^{2x}+e^x+c$

(f) $\int\frac{x^3-3x+4}{x}dx = \int\left(\frac{x^3}{x}-3\frac{x}{x}+\frac4x\right)dx = \int\left(x^2-3+\frac4x\right)dx = \frac{x^3}{3}-3x+4\ln x+c$

Section 10.2

5. (a) $\int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1^2}{2}-\frac02=\frac12$ (b) $\int_1^2(2x+x^2)dx = \left(x^2+\frac{x^3}{3}\right)\Big|_1^2 = 4+\frac83-1-\frac13 = 3+\frac73=\frac{16}{3}$

(c) $\int_{-2}^3\left(\frac12x^2-\frac13x^3\right)dx = \left(\frac16x^3-\frac1{12}x^4\right)\Big|_{-2}^3 = \frac16\cdot27-\frac1{12}\cdot81+\frac16\cdot8+\frac1{12}\cdot16 = \frac92-\frac{27}{4}+\frac43+\frac43=\ldots$

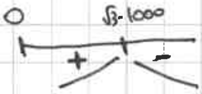
(d) $\int_0^2(t^3-t^4)dt = \left(\frac{t^4}{4}-\frac{t^5}{5}\right)\Big|_0^2 = \frac{16}{4}-\frac{32}{5} = \frac{80-128}{20} = -\frac{48}{20} = -\frac{12}{5}$

(e) $\int_1^2\left(2t^5-\frac1{t^2}\right)dt = \int_1^2(2t^5-t^{-2})dt = \left(2\frac{t^6}{6}-\frac{t^{-1}}{-1}\right)\Big|_1^2 = \left(\frac{t^6}{3}+\frac1t\right)\Big|_1^2 = \frac{64}{3}+\frac12-\frac13-1=\ldots$

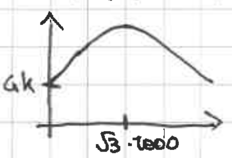
(f) $\int_2^3\left(\frac1{t-1}+t\right)dt = \left(\ln(t-1)+\frac{t^2}{2}\right)\Big|_2^3 = \ln2+\frac92-\ln1-\frac42 = \ln2+\frac52$

7. $f(x)=4000-x-\frac{3\cdot10^6}{x}$ (a) max profit $f'(x) = -1+\frac{3\cdot10^6}{x^2}$ $f'(x)=0 \Leftrightarrow \frac{3\cdot10^6}{x^2}=1$

$\Leftrightarrow x^2=3\cdot10^6 \Leftrightarrow x=\pm\sqrt3\cdot1000$. Since $x>0$



$f'(1)=-1+3\cdot10^6>0$ $f'(10^6)=-1+3\cdot10^{-6}<0$ So $x=\sqrt3\cdot1000$ maximises profit



(b) $I=\frac1{2000}\int_{1000}^{3000}f(x)dx = \frac1{2000}\left(4000x-\frac{x^2}{2}-3\cdot10^6\ln x\right)\Big|_{1000}^{3000} =$

$=\frac1{2000}\left(12\cdot10^6-\frac92-3\cdot10^6\ln3000-4\cdot10^6+\frac42+3\cdot10^6\ln1000\right)=$

$=\frac1{2000}\left(8\cdot10^6-\frac52-3\cdot10^6\ln3\right) = 4000-\frac5{4000}-\frac32\cdot1000\ln3 \approx 2352.08$

Section 10.3

(a) $\frac{d}{dt}\int_0^t x^2dx = t^2$ (b) $\frac{d}{dt}\int_t^3 e^{-x^2}dx = \frac{d}{dt}\int_3^t -e^{-x^2}dx = -e^{-t^2}$ (c) $\frac{d}{dt}\int_{-t}^t\frac1{\sqrt{x^4+1}}dx =$

$=\frac{d}{dt}\left(\int_0^t\frac1{\sqrt{x^4+1}}dx + \int_0^{-t}\frac1{\sqrt{x^4+1}}dx\right) = \frac1{\sqrt{t^4+1}}+\frac1{\sqrt{t^4+1}} = \frac2{\sqrt{t^4+1}}$

Section 10.4

2. (a) $f(r)=Br^{-2}$ there are n individuals in the population. The individuals with income in the interval $[b,2b]$ are $N=n\int_b^{2b}Br^{-2}dr = nB\frac{r^{-1}}{-1}\Big|_b^{2b} = nB\left(-\frac1{2b}+\frac1b\right) = n\frac{B}{2b}$

The total income of these people is $M=n\int_b^{2b}rBr^{-2}dr = n\int_b^{2b}Br^{-1}dr = nB\log r\Big|_b^{2b} =$

$=nB(\log2b-\log b) = nB\log2$. Then the mean is $m = \frac{M}{N} = 2b\log2$