Section 10.6

1. (a) $\int (x^2+1)^8 \, 2x \, dx$ sub: $u = x^2+1$, $\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$ $\Rightarrow \int u^8 \cdot \cancel{2x} \cdot \frac{du}{\cancel{2x}} = \int u^8 du = \frac{u^9}{9} + c = \frac{(x^2+1)^9}{9} + c$

(b) $\int (x+2)^{10} dx$ sub: $u = x+2$, $\frac{du}{dx} = 1 \Rightarrow dx = du$ $\Rightarrow \int u^{10} du = \frac{u^{11}}{11} + c = \frac{(x+2)^{11}}{11} + c$

(c) $\int \frac{2x-1}{x^2-x+8} dx$ sub: $u = x^2-x+8$, $\frac{du}{dx} = 2x-1 \Rightarrow dx = \frac{du}{2x-1}$ $\Rightarrow \int \frac{\cancel{2x-1}}{u} \cdot \frac{du}{\cancel{2x-1}} = \int \frac{1}{u} du = \ln u + c =$

$= \ln(x^2-x+8) + c$

2. (a) $\int x (2x^2+3)^5 dx$ sub: $u = 2x^2+3$, $\frac{du}{dx} = 4x \Rightarrow dx = \frac{du}{4x}$ $\Rightarrow \int \cancel{x} \underbrace{(2x^2+3)^5}_{=u} \cdot \frac{du}{4\cancel{x}} = \int u^5 \frac{du}{4} = \frac{1}{4} \cdot \frac{u^6}{6} + c = \frac{(2x^2+3)^6}{24} + c$

(b) $\int x^2 e^{x^3+2} dx$ sub: $u = x^3+2$, $\frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2}$ $\int \cancel{x^2} e^u \cdot \frac{du}{3\cancel{x^2}} = \frac{1}{3} \int e^u du = \frac{1}{3} e^u + c = \frac{1}{3} e^{x^3+2} + c$

(c) $\int \frac{\ln(x+2)}{2x+4}$ sub: $u = \ln(x+2)$, $\frac{du}{dx} = \frac{1}{x+2} \Rightarrow dx = du(x+2)$ $\int \frac{u}{2\cancel{(x+2)}} \cdot \cancel{(x+2)} du = \frac{1}{2} \int u du = \frac{1}{2} \frac{u^2}{2} + c =$

$= \frac{u^2}{4} + c = \frac{(\ln(x+2))^2}{4} + c$

(d) $\int x \sqrt{1+x} dx$ sub: $u = \sqrt{1+x}$, $\frac{du}{dx} = \frac{1}{2\sqrt{1+x}} \Rightarrow dx = 2\sqrt{1+x} \, du$

$\int x \cdot u \cdot 2\sqrt{1+x} \, du$ now since $u = \sqrt{1+x} \Rightarrow u^2 = 1+x \Rightarrow x = u^2 - 1$ so

$\int (u^2-1) \cdot u \cdot 2u \, du = \int 2u^2 (u^2-1) du = 2 \int (u^4 - u^2) du = 2\left(\frac{u^5}{5} - \frac{u^3}{3}\right) + c = 2\left(\frac{\sqrt{(1+x)^5}}{5} - \frac{\sqrt{(1+x)^3}}{3}\right) + c$

(e) $\int \frac{x^3}{(1+x^2)^3} dx$ $u = 1+x^2$, $\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$ $\int \frac{x^{\cancel{3}2}}{u^3} \cdot \frac{du}{2\cancel{x}} = \frac{1}{2} \int \frac{x^2}{u^3} du$ now $u = 1+x^2 \Rightarrow x^2 = u-1$

$\frac{1}{2} \int \frac{u-1}{u^3} du = \frac{1}{2} \int \left(\frac{1}{u^2} - \frac{1}{u^3}\right) du = \frac{1}{2}\left(\frac{u^{-1}}{-1} - \frac{u^{-2}}{-2}\right) + c = -\frac{1}{2u} + \frac{1}{4u^2} + c =$

$= -\frac{1}{2(1+x^2)} + \frac{1}{4(1+x^2)^2} + c$

(f) $\int x^5 \sqrt{4-x^3} dx$ $u = 4-x^3$, $\frac{du}{dx} = -3x^2 \Rightarrow dx = \frac{du}{-3x^2}$ $\int x^{\cancel{5}3} \sqrt{4-x^3} \cdot \frac{du}{-3\cancel{x^2}} = -\frac{1}{3} \int x^3 \sqrt{u} \, du$

now since $u = 4 - x^3 \Rightarrow x^3 = 4-u \Rightarrow -\frac{1}{3} \int (4-u)\sqrt{u} \, du = -\frac{1}{3} \int \left(4\sqrt{u} - u^{3/2}\right) du =$

$= -\frac{1}{3}\left(4 \cdot \frac{u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2}\right) + c = -\frac{1}{3}\left(\frac{8}{3} u^{\frac{3}{2}} - \frac{2}{5} u^{\frac{5}{2}}\right) + c = -\frac{1}{3}\left(\frac{8}{3}(4-x^3)^{\frac{3}{2}} - \frac{2}{5}(4-x^3)^{\frac{5}{2}}\right) + c$

3. (a) $\int_0^1 x\sqrt{1+x^2} dx$ $u = 1+x^2$, $\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$ $\Rightarrow \int \cancel{x} \sqrt{u} \frac{du}{2\cancel{x}} = \frac{1}{2} \int \sqrt{u} du = \frac{1}{\cancel{2}} \frac{u^{\frac{3}{2}}}{3/\cancel{2}} + c = \frac{u\sqrt{u}}{3} + c =$

$= \frac{(1+x^2)\sqrt{1+x^2}}{3} + c \Rightarrow \int_0^1 x\sqrt{1+x^2} dx = \frac{(1+x^2)\sqrt{1+x^2}}{3}\Big|_0^1 = \frac{2\sqrt{2}}{3} - \frac{1}{3} = \frac{2\sqrt{2}-1}{3}$

(b) $\int_1^e \frac{\ln x}{x} dx$ $u = \ln x$, $\frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x \, du$ $\int \frac{u}{\cancel{x}} \cancel{x} du = \int u du = \frac{u^2}{2} + c = \frac{(\ln x)^2}{2} + c$

$\int_1^e \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2}\Big|_1^e = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$

(c) $\int_1^3 \frac{1}{x^2} e^{2/x} dx$ $u = 2/x$, $\frac{du}{dx} = -\frac{2}{x^2} \Rightarrow dx = -\frac{x^2}{2} du$ $\int \frac{1}{\cancel{x^2}} e^u \cdot \left(-\frac{\cancel{x^2}}{2}\right) du = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + c =$

$= -\frac{1}{2} e^{2/x} + c$ $\int_1^3 \frac{1}{x^2} e^{2/x} dx = -\frac{1}{2} e^{2/x}\Big|_1^3 = -\frac{1}{2} e^{2/3} + \frac{1}{2} e^2$

(d) $\int_5^8 \frac{x}{x-4}\,dx$ $\quad u = x-4$, $\frac{du}{dx} = 1 \Rightarrow du = dx$ $\quad$ also $u = x-4 \Rightarrow x = u+4$

So $\int \frac{u+4}{u}\,du = \int \left(1+\frac{4}{u}\right)du = u + 4\ln u + c = x-4+4\ln(x-4)+c = x+4\ln(x-4)+c$

$$\int_5^8 \frac{x}{x-4}\,dx = \left(x+4\ln(x-4)\right)\Big|_5^8 = 8+4\ln 4 - 5 - 4\cancel{\ln 1} = 3+4\ln 4 = 3+8\ln 2$$

Other method: $\int \frac{x}{x-4}\,dx = \int \frac{x-4+4}{x-4}\,dx = \int \left(\frac{\cancel{x-4}}{\cancel{x-4}} + \frac{4}{x-4}\right)dx = x + 4\ln(x-4)+c$

Section 10.7

1. (a) $\int_1^\infty \frac{1}{x^3}\,dx = \frac{x^{-2}}{-2}\Big|_1^\infty = -\frac{1}{2x^2}\Big|_1^\infty = \lim_{x\to+\infty}\left(-\frac{1}{2x^2}\right) - \left(-\frac{1}{2\cdot 1}\right) = 0+\frac{1}{2} = \frac{1}{2}$

(b) $\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \frac{x^{\frac{1}{2}}}{1/2}\Big|_1^\infty = 2\sqrt{x}\Big|_1^\infty = \lim_{x\to+\infty}(2\sqrt{x}) - 2\sqrt{1} = +\infty$

(c) $\int_{-\infty}^0 e^x\,dx = e^x\Big|_{-\infty}^0 = e^0 - \lim_{x\to-\infty} e^x = 1-0 = 1$

(d) $\int_0^a \frac{x}{\sqrt{a^2-x^2}}\,dx$, $a>0$ $\quad u = a^2-x^2$, $\frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$ $\quad \int \frac{\cancel{x}}{\sqrt{u}}\cdot\frac{du}{-2\cancel{x}} = -\frac{1}{2}\int u^{-\frac{1}{2}}\,du =$

$= -\frac{1}{\cancel{2}}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{\cancel{2}}}\right)+c = -\sqrt{u}+c = -\sqrt{a^2-x^2}+c$

$$\int_0^a \frac{x}{\sqrt{a^2-x^2}}\,dx = \left(-\sqrt{a^2-x^2}\right)\Big|_0^a = \lim_{x\to a^-}\left(-\sqrt{a^2-x^2}\right) - \left(-\sqrt{a^2-0^2}\right) = 0+\sqrt{a^2} = a$$