## Section 14.1

1. $f(x,y)=x+2y \quad f(0,1)=0+2\cdot 1=2 \quad f(2,-1)=2+2\cdot(-1)=2-2=0 \quad f(a,a)=a+2a=3a$

$f(a+h,b)-f(a,b)=\cancel{a}+h+\cancel{2b}-\cancel{a}-\cancel{2b}=h$

3. $f(x,y)=3x^2-2xy+y^3. \quad f(1,1)=3\cdot 1^2-2\cdot 1\cdot 1+1^3=3-2+1=2 \quad f(-2,3)=3\cdot(-2)^2-2\cdot(-2)\cdot 3+3^3=$

$=12+12+27=51 \quad f(1/x,1/y)=3\cdot\frac{1}{x^2}-2\cdot\frac{1}{x}\cdot\frac{1}{y}+\frac{1}{y^3}=\frac{3}{x^2}-\frac{2}{xy}+\frac{1}{y^3}$

$$p=\frac{f(x+h,y)-f(x,y)}{h}=\frac{3(x+h)^2-2(x+h)y+y^3-3x^2+2xy-y^3}{h}=\frac{\cancel{3x^2}+6xh+3h^2-\cancel{2xy}-2hy+\cancel{y^3}-\cancel{3x^2}+\cancel{2xy}-\cancel{y^3}}{h}=$$

$$=\frac{6xh+3h^2-2hy}{h}=6x+3h-2y$$

$$q=\frac{f(x,y+k)-f(x,y)}{k}=\frac{3x^2-2x(y+k)+(y+k)^3-3x^2+2xy-y^3}{k}=\frac{\cancel{3x^2}-\cancel{2xy}-2xk+\cancel{y^3}+3y^2k+3yk^2+k^3-\cancel{3x^2}+\cancel{2xy}-\cancel{y^3}}{k}=$$

$$=\frac{-2xk+3y^2k+3yk^2+k^3}{k}=-2x+3y^2+3yk+k^2$$

## Section 14.2

2. (a) $z=x^2+3y^2 \quad \frac{\partial z}{\partial x}=2x \quad \frac{\partial z}{\partial y}=6y$ (b) $z=xy \quad \frac{\partial z}{\partial x}=y \quad \frac{\partial z}{\partial y}=x$

(c) $z=5x^4y^2-2xy^5 \quad \frac{\partial z}{\partial x}=5\cdot 4x^3y^2-2y^5=20x^3y^2-2y^5 \quad \frac{\partial z}{\partial y}=5x^4\cdot 2y-2x\cdot 5y^4=10x^4y-10xy^4$

(d) $z=e^{x+y} \quad \frac{\partial z}{\partial x}=e^{x+y} \quad \frac{\partial z}{\partial y}=e^{x+y}$ (e) $z=e^{xy} \quad \frac{\partial z}{\partial x}=e^{xy}\cdot y \quad \frac{\partial z}{\partial y}=e^{xy}\cdot x$

(f) $z=e^x/y \quad \frac{\partial z}{\partial x}=\frac{e^x}{y} \quad \frac{\partial z}{\partial y}=-\frac{e^x}{y^2}$ (g) $z=\ln(x+y) \quad \frac{\partial z}{\partial x}=\frac{1}{x+y} \quad \frac{\partial z}{\partial y}=\frac{1}{x+y}$

(h) $z=\ln(xy) \quad \frac{\partial z}{\partial x}=\frac{1}{xy}\cdot y=\frac{1}{x} \quad \frac{\partial z}{\partial y}=\frac{1}{xy}\cdot x=\frac{1}{y}$

3. (a) $f(x,y)=x^7-y^7 \quad f'_1(x,y)=7x^6 \quad f'_2(x,y)=-7y^6 \quad f''_{12}(x,y)=0$

(b) $f(x,y)=x^5\ln y \quad f'_1(x,y)=5x^4\ln y \quad f'_2(x,y)=x^5/y \quad f'_{12}(x,y)=5\frac{x^4}{y}$

(c) $f(x,y)=(x^2-2y^2)^5 \quad f'_1(x,y)=5(x^2-2y^2)^4\cdot 2x=10x(x^2-2y^2)^4 \quad f'_2(x,y)=5(x^2-2y^2)^4\cdot(-4y)=$

$=-20y(x^2-2y^2)^4 \quad f'_{12}(x,y)=10x\cdot 4(x^2-2y^2)^3\cdot(-4y)=-160xy(x^2-2y^2)^3$

## Section 14.4

2. (a) distance between $(-1,2,3)$ and $(4,-2,0)$ $\quad d=\sqrt{(4-(-1))^2+(-2-2)^2+(0-3)^2}=$

$=\sqrt{25+16+9}=\sqrt{50}=5\sqrt{2}$ (b) $(a,b,c)$ and $(a+1,b+1,c+1)$ $\quad d=\sqrt{(\cancel{a}+1-\cancel{a})^2+(\cancel{b}+1-\cancel{b})^2+(\cancel{c}+1-\cancel{c})^2}=\sqrt{3}$

3. center $(2,1,1)$, sphere radius 5: $(x-2)^2+(y-1)^2+(z-1)^2=25$

4. $(x+3)^2+(y-3)^2+(z-4)^2=25$ is the equation of a sphere with ~~center~~ $(-3,3,4)$ and radius 5.

## Section 14.5

1. $f(x,y,z)=xy+xz+yz.$ (a) $f(-1,2,3)=-1\cdot 2+(-1)\cdot 3+2\cdot 3=-2-3+6=1$

$f(a+1,b+1,c+1)-f(a,b,c)=(a+1)(b+1)+(a+1)(c+1)+(b+1)(c+1)-ab-ac-bc=$

$=\cancel{ab}+b+a+1+\cancel{ac}+a+c+1+\cancel{bc}+b+c+1-\cancel{ab}-\cancel{ac}-\cancel{bc}=2a+2b+2c+3$

(b) $f(tx,ty,tz)=(tx)\cdot(ty)+(tx)\cdot(tz)+(ty)\cdot(tz)=t^2xy+t^2xz+t^2yz=t^2(xy+xz+yz)=t^2f(x,y,z)$

## Section 14.9

1. $M=0.14Y+76.03(r-2)^{-0.86}, \ r>2 \quad \frac{\partial M}{\partial Y}=0.14 \quad \frac{\partial M}{\partial r}=76.03\cdot(-0.86)(r-2)^{-1.86}=-63.87(r-2)^{-1.86}$

$\frac{\partial M}{\partial Y}>0 \Rightarrow M$ increases with increasing of $Y$ $\quad \frac{\partial M}{\partial r}<0 \Rightarrow M$ decreases with increasing of $r$.