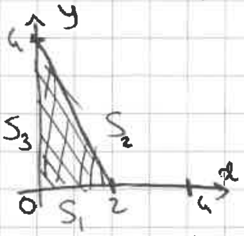
we have $-\frac{b}{3a}=\frac{2}{3}$ and $-\frac{b}{6}=\frac{1}{3} \Rightarrow b=-2 \qquad \frac{2}{3a}=\frac{2}{3} \Rightarrow a=1$

$f(\frac{2}{3},\frac{1}{3})=\frac{1}{-9} \Rightarrow a\cdot\frac{4}{9}\cdot\frac{1}{3}+b\cdot\frac{2}{3}\cdot\frac{1}{3}+2\cdot\frac{2}{3}\cdot\frac{1}{9}+c=-\frac{1}{9}$

$\Rightarrow \frac{4}{27}-\frac{4}{9}+\frac{4}{27}+c=-\frac{1}{9} \Rightarrow c=\frac{4}{9}-\frac{1}{9}-\frac{8}{27}=\frac{1}{27}$

$f(x,y)=x^2y-2xy+2xy^2+\frac{1}{27}$

(b) $S=\{(x,y): x\geq 0,\ y\geq 0,\ 2x+y\leq 4\}$



Interior: As we saw before, crit. points are $(0,0)$, $(-\frac{b}{a},0)=(2,0)$, $(0,-\frac{b}{2})=(0,1)$ and $(\frac{2}{3},\frac{1}{3})$.

Boundary: S_1: $y=0$, $0\leq x\leq 2$ $\qquad g_1(x)=\frac{1}{27}$ (constant) $\Rightarrow g_1'(x)=0$

S_2: $2x+y=4$, $0\leq x\leq 2$, $0\leq y\leq 4$. $\qquad y=4-2x \Rightarrow g_2(x)=x^2(4-2x)-2x(4-2x)+$

$+2x\underbrace{(4-2x)^2}_{16+4x^2-16x}+\frac{1}{27}=4x^2-2x^3-8x+4x^2+32x+8x^3-32x^2+\frac{1}{27}=6x^3-24x^2+24x+\frac{1}{27}$

$g_2'(x)=18x^2-48x+24=6(3x^2-8x+4) \qquad g_2'(x)=0 \Leftrightarrow 3x^2-8x+4=0 \qquad x=\frac{4\pm\sqrt{16-12}}{3}=$

$=\frac{4\pm 2}{3}=\begin{cases} 6/3=2 \to y=4-4=0 \\ 2/3 \to y=4-2\cdot\frac{2}{3}=8/3 \end{cases}$ we add $(\frac{2}{3},\frac{8}{3})$ and $(0,4)$

S_3: $x=0$, $0\leq y\leq 4 \qquad g_3(y)=\frac{1}{27}$ constant

Candidates: $(0,0)$, $(2,0)$, $(0,1)$, $(\frac{2}{3},\frac{1}{3})$, $(\frac{2}{3},\frac{8}{3})$

$f(0,0)=f(2,0)=f(0,1)=\frac{1}{27} \qquad f(\frac{2}{3},\frac{1}{3})=-\frac{1}{9} \qquad f(\frac{2}{3},\frac{8}{3})=$

$=\frac{4}{9}\cdot\frac{8}{3}-2\cdot\frac{2}{3}\cdot\frac{8}{3}+2\cdot\frac{2}{3}\cdot\frac{64}{9}+\frac{1}{27}=\frac{32}{27}-\frac{32}{9}+\frac{256}{27}+\frac{1}{27}=\frac{193}{27}$

MIN at $(\frac{2}{3},\frac{1}{3})$, MAX at $(\frac{2}{3},\frac{8}{3})$

Section 11.4

2. (a) $\frac{1}{5}+(\frac{1}{5})^2+(\frac{1}{5})^3+\ldots=\frac{1}{5}(1+\frac{1}{5}+(\frac{1}{5})^2+\ldots)=\frac{1}{5}\sum_{n=0}^{\infty}(\frac{1}{5})^n=\frac{1}{5}\cdot\frac{1}{1-\frac{1}{5}}=\frac{1}{\cancel{5}}\cdot\frac{\cancel{5}}{4}=\frac{1}{4}$

(b) $0.1+(0.1)^2+(0.1)^3+\ldots=0.1(1+0.1+(0.1)^2+\ldots)=0.1\sum_{n=0}^{\infty}(0.1)^n=0.1\cdot\frac{1}{1-0.1}=\frac{1}{10}\cdot\frac{10}{9}=\frac{1}{9}$

(c) $517+517(1.1)^{-1}+517(1.1)^{-2}+\ldots=517\sum_{n=0}^{\infty}(1.1)^{-n}=517\cdot\frac{1}{1-(1.1)^{-1}}=5687$

(d) $a+a(1+a)^{-1}+a(1+a)^{-2}+\ldots=a\sum_{n=0}^{\infty}(1+a)^{-n}=a\cdot\frac{1}{1-(1+a)^{-1}}=a\cdot\frac{1}{1-\frac{1}{1+a}}=\cancel{a}\cdot\frac{a+1}{\cancel{a}}=a+1$

(e) $5+\frac{5\cdot 3}{7}+\frac{5\cdot 3^2}{7^2}+\ldots=5\cdot\sum_{n=0}^{\infty}(\frac{3}{7})^n=5\cdot\frac{1}{1-\frac{3}{7}}=5\cdot\frac{7}{4}=\frac{35}{4}$

3. (a) $8+1+\frac{1}{8}+\frac{1}{64}+\ldots=8(1+\frac{1}{8}+\frac{1}{64}+\ldots)=8\cdot\sum_{n=0}^{\infty}(\frac{1}{8})^n=8\cdot\frac{1}{1-\frac{1}{8}}=8\cdot\frac{8}{7}=\frac{64}{7}$

(b) $-2+6-18+54-\ldots$ not geometric $\qquad$ (c) $2^{1/3}+1+2^{-1/3}+2^{-2/3}+\ldots=2^{1/3}(1+2^{-1/3}+2^{-2/3}+2^{-1}+\ldots)=$

$=2^{1/3}\sum_{n=0}^{\infty}(2^{-1/3})^n=2^{1/3}\cdot\frac{1}{1-2^{-1/3}}=2^{1/3}\cdot\frac{2^{1/3}}{2^{1/3}-1}=\frac{2^{2/3}}{2^{1/3}-1}$ $\qquad$ (d) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ not geometric

4. (a) $\frac{1}{p}+\frac{1}{p^2}+\frac{1}{p^3}+\ldots=\frac{1}{p}\sum_{n=0}^{\infty}\frac{1}{p^n}=\frac{1}{p}\cdot\frac{1}{1-\frac{1}{p}}=\frac{1}{\cancel{p}}\cdot\frac{\cancel{p}}{p-1}=\frac{1}{p-1}$ converges if $p>1$

(b) $x+\sqrt{x}+1+\frac{1}{\sqrt{x}}+\ldots=x(1+\frac{1}{\sqrt{x}}+\frac{1}{x}+\frac{1}{x^{3/2}}+\ldots)=x\sum_{n=0}^{\infty}(\frac{1}{\sqrt{x}})^n=x\cdot\frac{1}{1-\frac{1}{\sqrt{x}}}=\frac{x\sqrt{x}}{\sqrt{x}-1}$ $\qquad$ (c) $\sum_{n=1}^{\infty}x^{2n}=\frac{1}{1-x^2}$