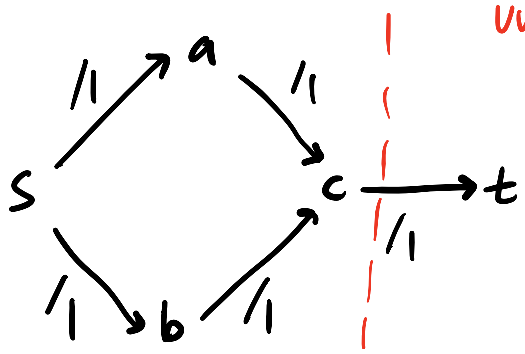
Uniqueness : Prove/disprove for flow network G

(a) G has unique min-cut $\Rightarrow$ G has unique max flow

(b) G has unique max flow $\Rightarrow$ G has unique min-cut

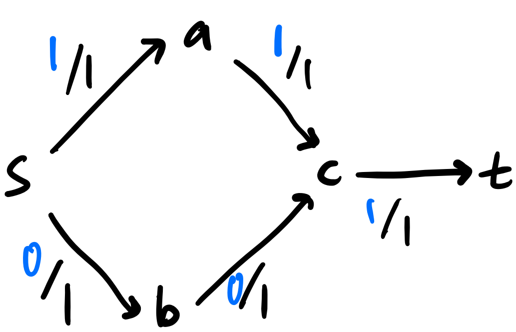
(a)

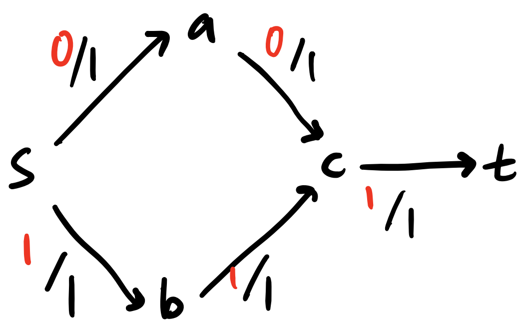


unique min cut

$\{s,a,b,c\}$ & $\{t\}$

However: two distinct max flows



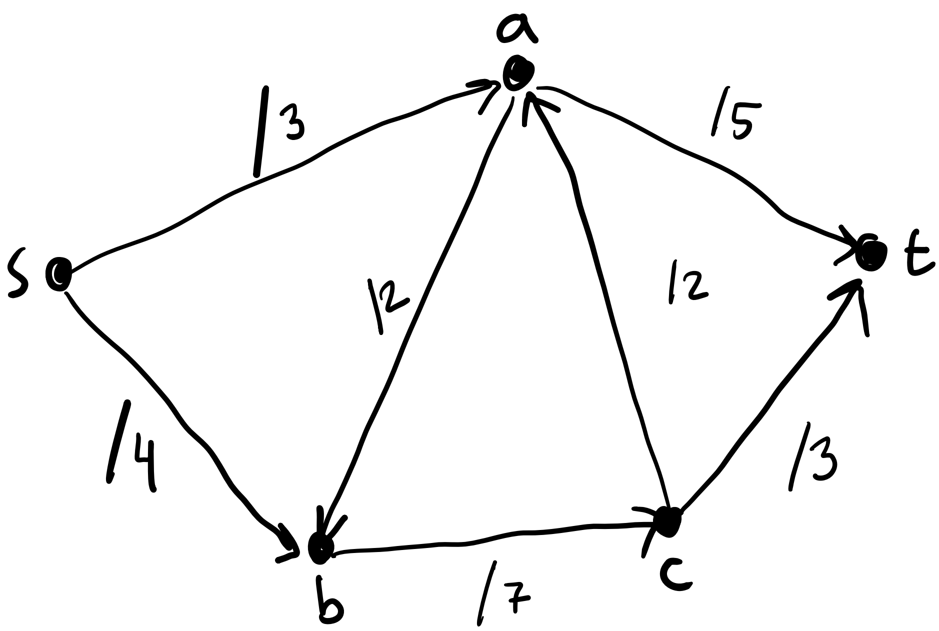


(b) $s \xrightarrow{/1} a \xrightarrow{/1} t$

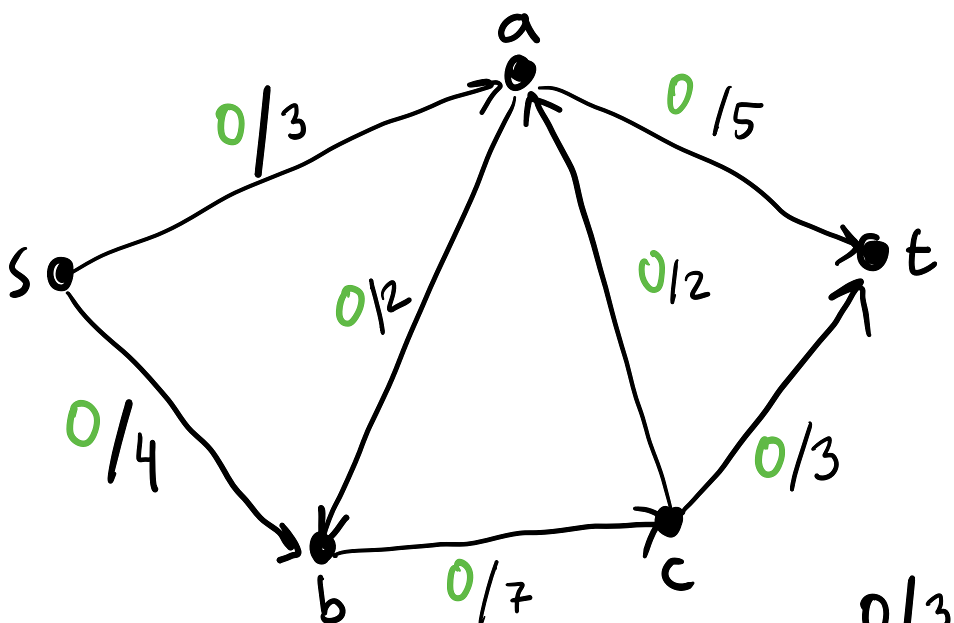
has a unique max flow but two min cuts

s|at resp sa|t

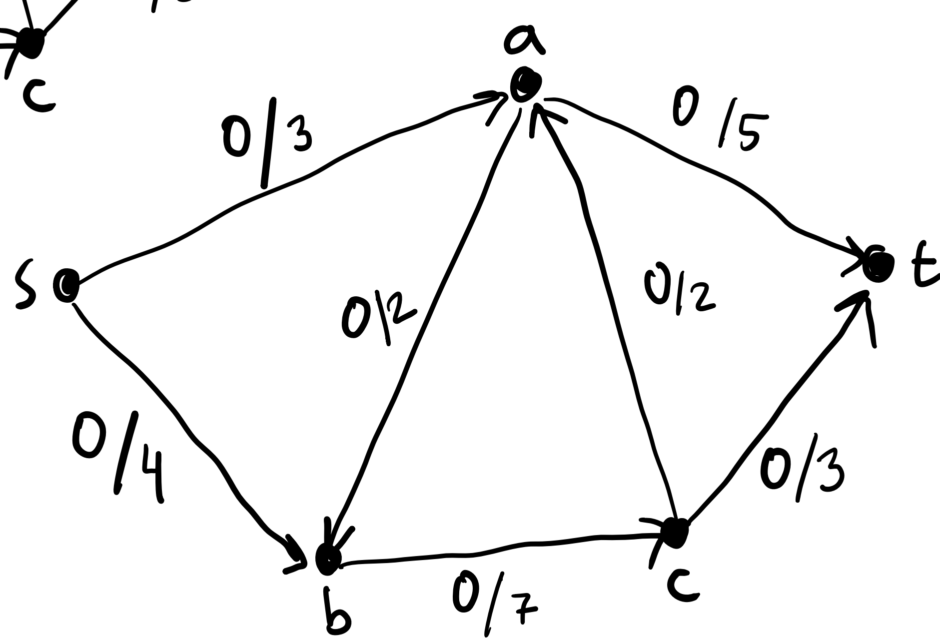
Flow network with capacities. Apply Ford-Fulkerson.



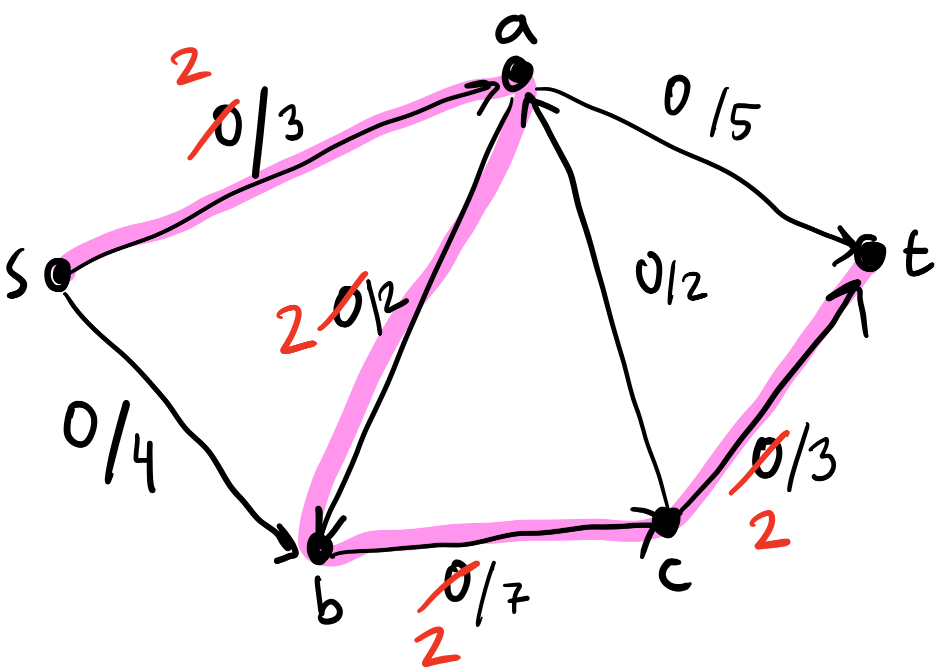
Initialize zero-flow:



First residual:



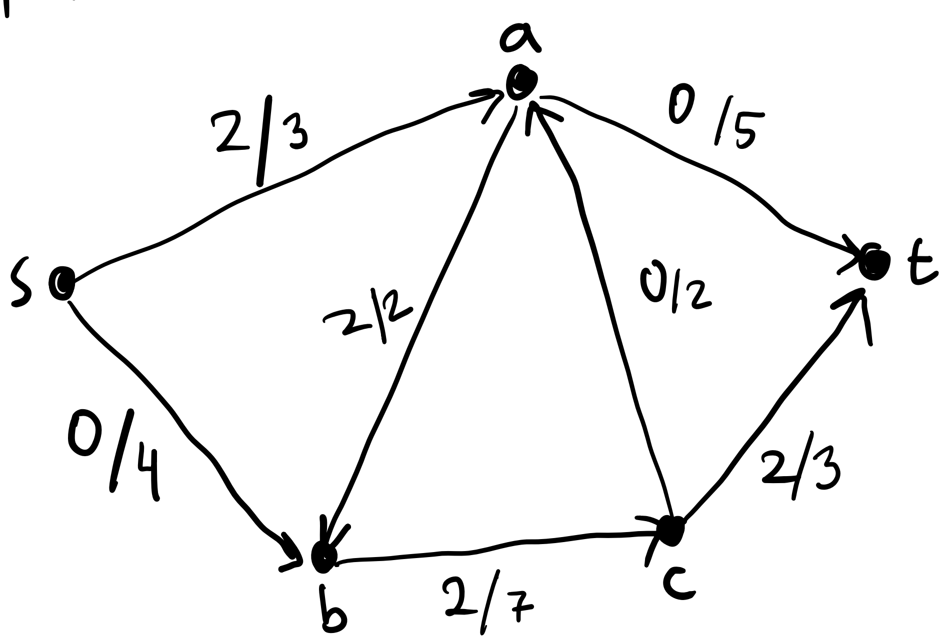
augmenting path 1



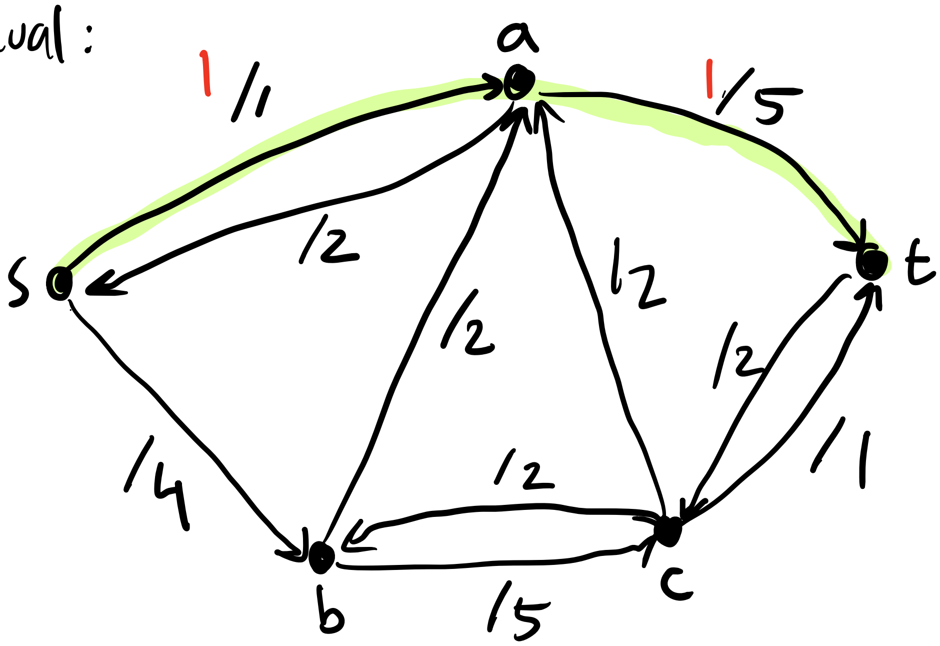
$$c_f = \min\{c_f(u,v) : (u,v) \in P\}$$

$$= \{3, 2, 7, 3\} = 2$$

update flow in network



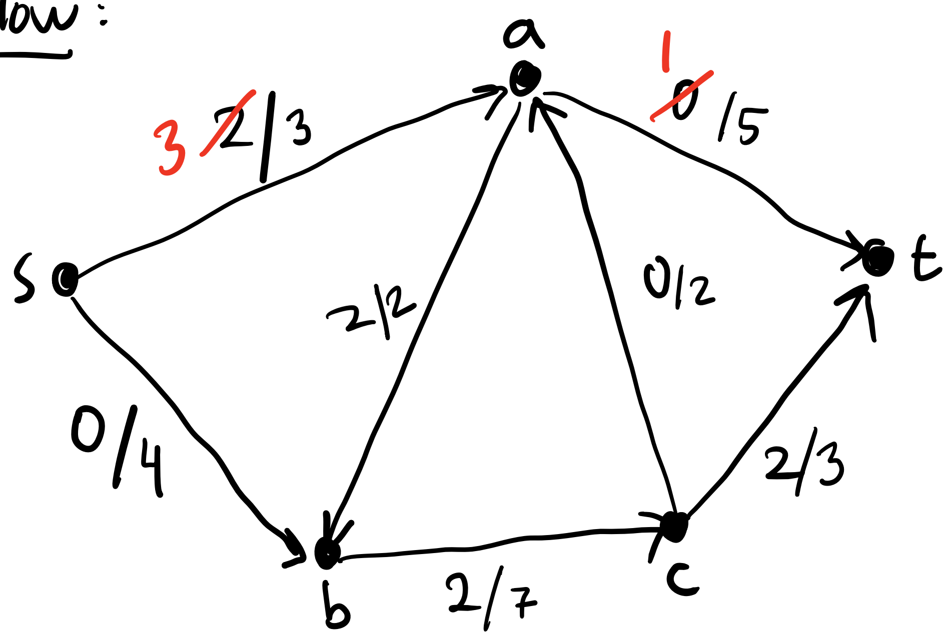
Residual:



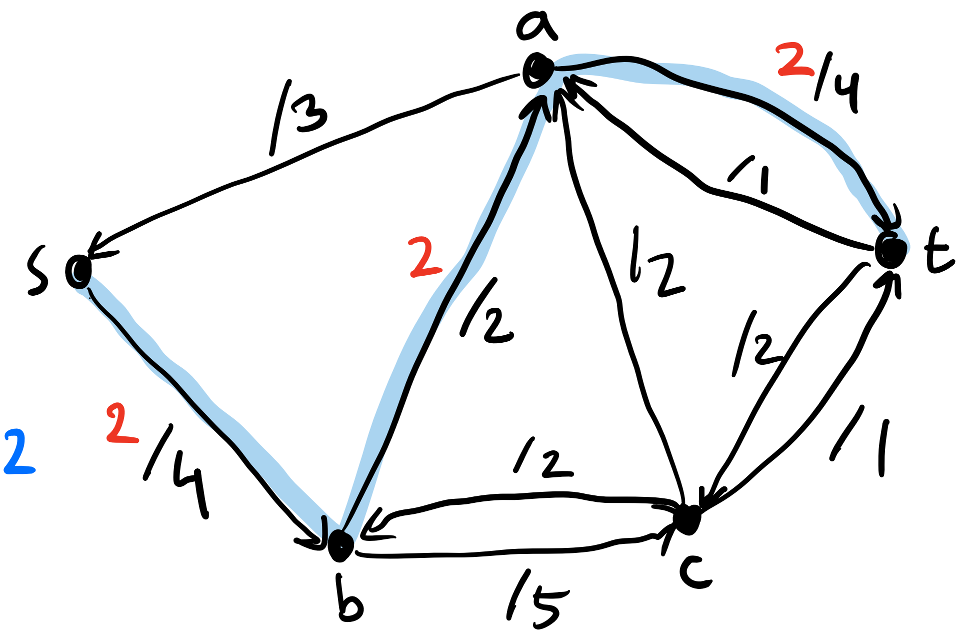
augmenting path capacity

min{1,5}=1

Updated flow:

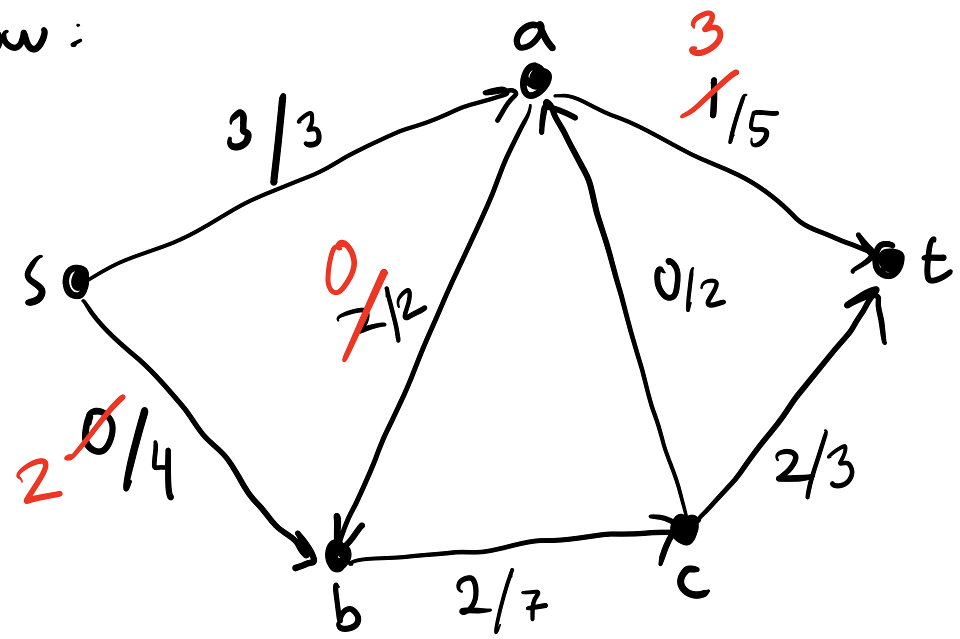


residual:

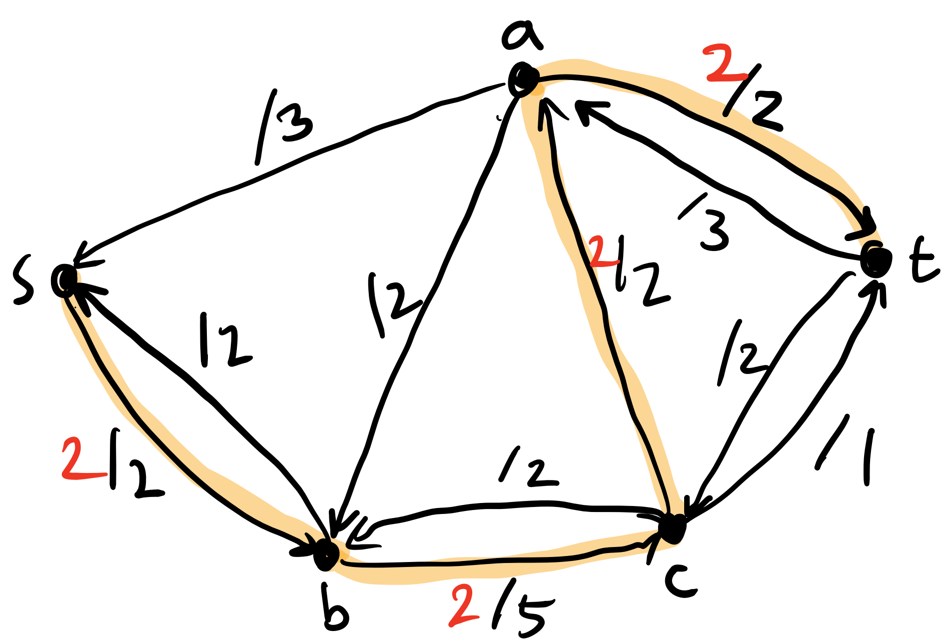


min{4,2,4}=2

New flow:

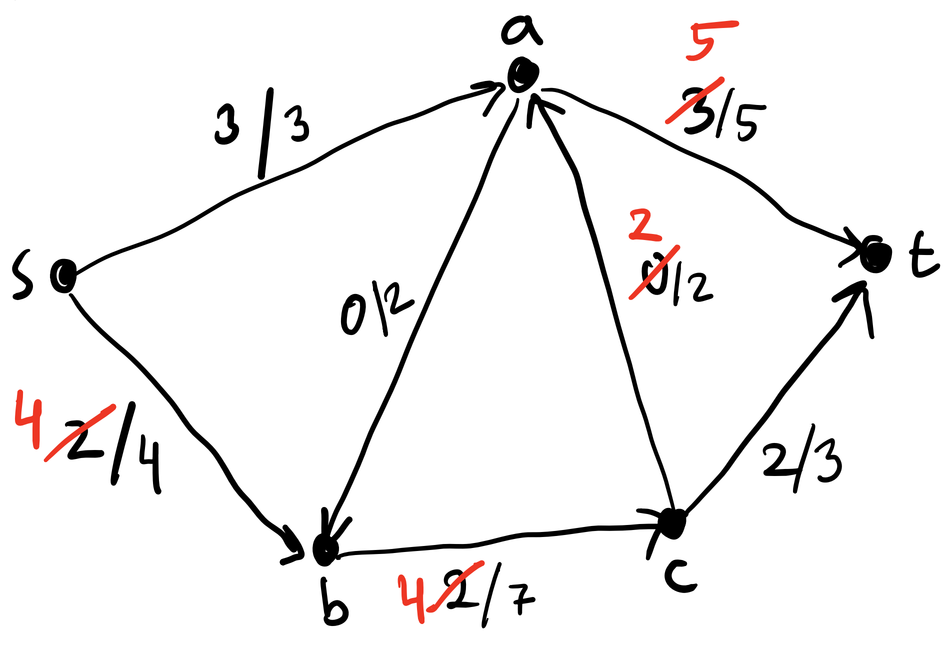


residual:

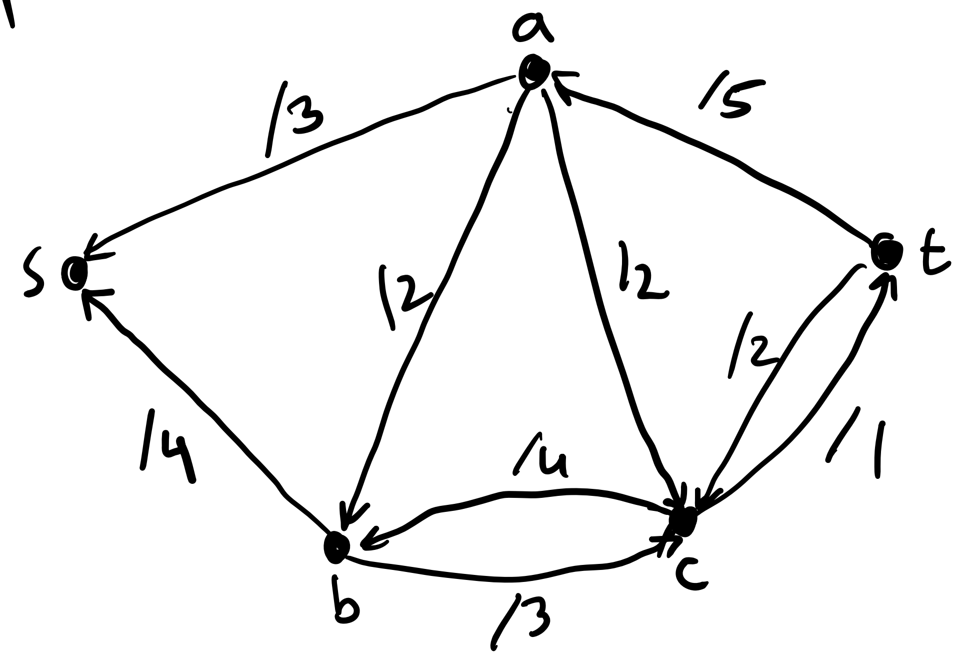


min{2, 5, 2, 2}
= 2

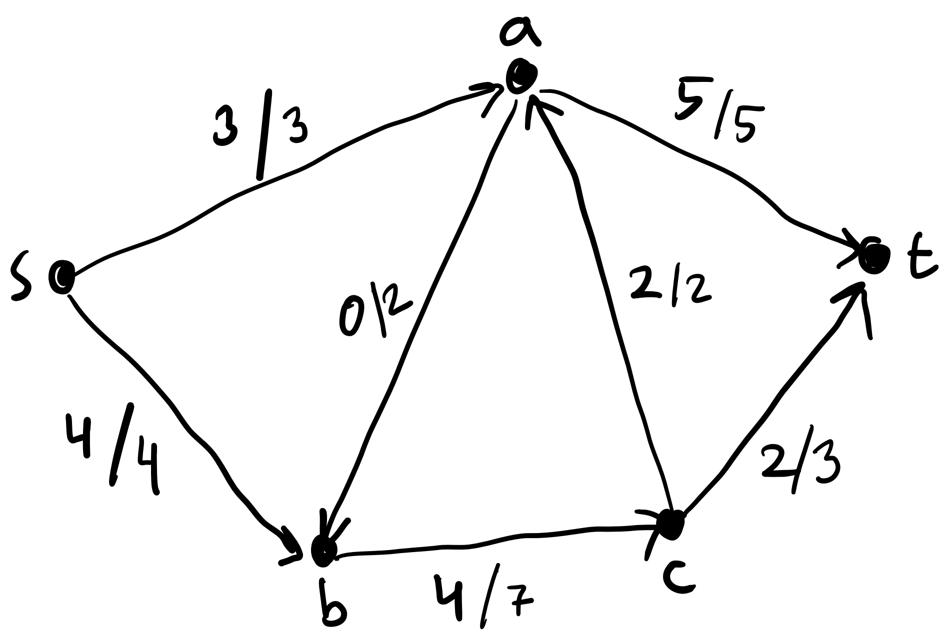
New flow:



Residual



for sure no paths left! Terminate

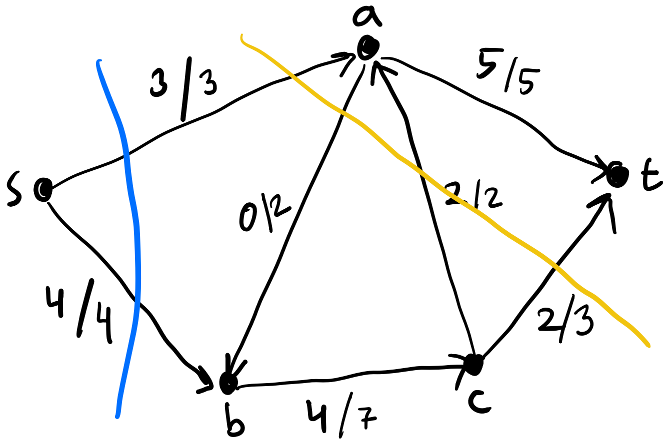


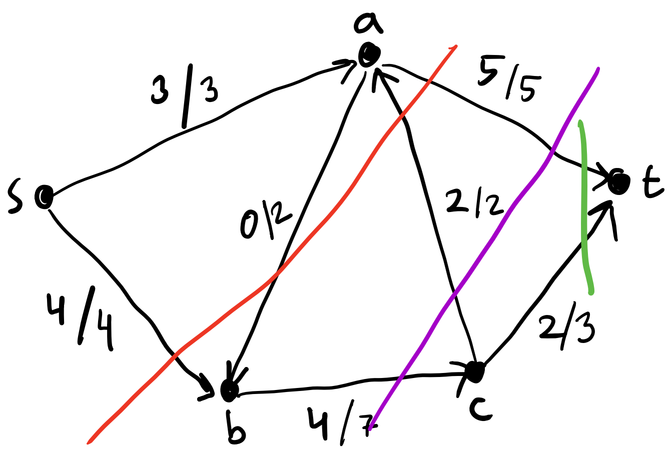
Sanity check:

(F1) $0 \le f(u,v) \le c(u,v) \quad \forall\, u,v \in V$

(F2) $\underbrace{\sum_{u\in V} f(u,v)}_{\text{inflow}} = \underbrace{\sum_{u \in V} f(v,u)}_{\text{outflow}} \quad \forall\, v \in V$

$$|f| = \sum_{v \in V} f(s,v) - \sum_{v \in V} f(v,s) = (3+4) - 0 = 7$$





cut = 7
(min)

4 + 5 = 9

4 + 5 = 9

3 + 2 + 2 = 7

5 + 2 = 7

Many min-cuts! Also many max-flows