Topology, Fall 2025

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Homework exercise #1

Due: Thursday Sept 18, 2025

Instructions: We strongly prefer that you typeset your answers with Latex or similar. Handwritten assignments will be graded only if reasonably easy to read.

You may work together on the assignment, but write your own solution and do not copy. You also are welcome to consult the instructors, after you made a decent effort on a problem. Tell us what you have tried, and we will be happy to provide some direction if needed.

Please, please, do not seek inappropriate help on the internet or from ChatGPT and the like.

Let $\mathbb{N}$ denote the set of positive integers. Let $\mathcal{T}$ be the collection of subsets U of $\mathbb{N}$ that satisfy the following condition: if x is in U and x is odd then $x+1$ is in U.

1. Show that $\mathcal{T}$ is a topology on $\mathbb{N}$.
2. Find a minimal basis for $\mathcal{T}$. I.e., find a collection $\mathcal{B}$ of subsets of $\mathbb{N}$ such that $\mathcal{B}$ is a basis for $\mathcal{T}$, but no proper subset of $\mathcal{B}$ is a basis.
3. Let $A = \{2, 3, 4, 5\}$. Find the interior and the closure of A in $\mathcal{T}$.
4. Let $\mathbb{N}_{\mathcal{T}}$ denote the set of positive integers, considered as a topological space with the topology $\mathcal{T}$. Let $f\colon \mathbb{N}_{\mathcal{T}} \to \mathbb{N}_{\mathcal{T}}$ be the function $f(x) = x^2$. Is f a continuous function?
5. The same as part (4) with $f(x) = \lfloor \sqrt{x} \rfloor$.