Topology, Fall 2025

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Homework exercise #2
Due: Thursday Oct 2, 2025

Instructions: We strongly prefer that you typeset your answers with Latex or similar. Handwritten assignments will be graded only if reasonably easy to read.

You may work together on the assignment, but write your own solution and do not copy. You also are welcome to consult the instructors, after you made a decent effort on a problem. Tell us what you have tried, and we will be happy to provide some direction if needed.

Please, please, do not seek inappropriate help on the internet or from ChatGPT and the like.

1. Suppose we have continuous maps $X \overset{s}{\underset{r}{\rightleftarrows}} Y$ such that $r \circ s$ is the identity map on Y.

   (a) Prove that r is a quotient map.

   (b) Prove that s is an embedding.

2. Define $\sim$ to be the following equivalence relation on $\mathbb{R}^2$: $(x, y) \sim (x', y')$ if $xy = x'y'$. Let $X = \mathbb{R}^2/_\sim$ be the quotient space of $\mathbb{R}^2$ by this equivalence relation.

   (a) Show that there is a continuous bijection from X to $\mathbb{R}$. Is this bijection a homeomorphism?

   (b) Is the quotient map $\mathbb{R}^2 \twoheadrightarrow X$ an open map?

   (c) Is it a closed map?