Topology, Fall 2025

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Homework exercise #3
Due: Thursday Oct 16, 2025

Instructions: We strongly prefer that you typeset your answers with Latex or similar. Handwritten assignments will be graded only if easy to read.

You may work together on the assignment, but write your own solution and do not copy. You also are welcome to consult the instructors, after you made a decent effort on a problem. Tell us what you have tried, and we will be happy to provide some direction if needed.

Please, please, do not seek inappropriate help on the internet or from ChatGPT and the like.

1. Let X be a connected topological space and let $A \subset X$. Assume that the boundary of A is connected. Show that also the closure of A is connected. (20 points)
2. Let X be a compact Hausdorff space and $f\colon X \to X$ a continuous map. Show that there is a nonempty closed set $F \subset X$ such that $f(F) = F$. (Hint: consider the sequence $F_1 = f(X)$, $F_{n+1} = f(F_n)$.) (15 points)
3. Let $\mathbb{R}$ be the real line with the standard topology, and let $\mathbb{Z} \subset \mathbb{R}$ be the set of integers. Let $\sim$ be the equivalence relation on $\mathbb{R}$ whose only non-singleton equivalence class is $\mathbb{Z}$. Let $\mathbb{R}/_\sim$ be the quotient space of $\mathbb{R}$ by this relation. Prove that $\mathbb{R}/_\sim$ is Hausdorff, but not locally compact. (15 points)