STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam: Introduction to Finance Mathematics (MT5009), 2026-05-25

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Allowed aid: Calculator (provided by the department).

Return of exam: To be announced via the course webpage or the course forum.

The exam consists of five problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Any assumptions should be clearly stated and motivated.
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.
- Write your code number (but no name) on each sheet.

Preliminary grading:

A	B	C	D	E
46	41	36	30	25

Good luck!

Problem 1

Consider a bond with face value $F = 200$ maturing in 3 years. The bond pays annual coupons $C = 10$. The first coupon is paid 1 year from now. The continuously compounded interest rate is $r = 0.05$.

(A) Find the present value of the bond.

(B) Find the value of the bond immediately after the first coupon is paid.

(10p)

Problem 2

Let us here consider the one-period binomial model ($N = 1$). The current share price is $S(0)$. The share price at time 1 will be either $S(0)(1+U)$ or $S(0)(1+D)$, where the probability of an up-jump is $p \in (0, 1)$. The risk-free interest rate (periodic compounding) is R. Suppose

$$R = 0.05, \qquad U = 0.2, \qquad D = -0.1, \qquad S(0) = 5.$$

Consider also a derivative with maturity at time 1 and payoff function

$$f(s) = \max(s^2 - 4s, 0).$$

(A) Determine the value of the derivative assuming that it is European.

(B) Determine the value of the derivative assuming that it is American. Also, determine the associated replicating portfolio.

(10p)

Problem 3

Consider two risky assets 1 and 2 with expected returns

$$m = (\mu_1, \mu_2) = (\mu, \mu)$$

where $\mu > 0$ (i.e., the two assets have the same expected returns) and covariance matrix

$$C = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}$$

Assume that there also exists a risk-free asset with interest rate $R = 0$.

State the mathematical problem that corresponds to finding the *market portfolio*, and solve it.

(10p)

Problem 4

In this question we consider the binomial model parameters:

$$R = 0.05, \quad U = 0.1, \quad D = -0.1, \quad S(0) = 100.$$

(A) Consider a two-period binomial model and a European call option with maturity at $t = 2$ and strike price $X = 110$. Find the value of the option.

(B) Consider a four-period binomial model and a European call option with maturity at $t = 4$ and strike price $X = 140$. Find the value of the option.

(10p)

Problem 5

Consider a non-dividend paying asset with price process $S(t), t \geq 0$. The continuously compounded risk-free interest rate is a constant r.

In the market there is forward contract with forward price $F(0,T)$ and a European call option: both with maturity at a date $T > 0$ and the asset as underlying. The strike price of the option equals the forward price i.e., $X = F(0,T)$.

(A) Show how the forward price $F(0,T)$ can be expressed in terms of $S(0)$, r, and T.

(B) State the payoffs of the forward and the call option.

Consider a fixed time $t \in (0,T)$. Can we determine if one of the two derivatives has a greater (or equal) no-arbitrage value compared to the other at t, and if so which has the greater value? (Do not forget to motivate your answer).

(10p)