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Suggested solutions

Exam: Introduction to Finance Mathematics (MT5009), 2026-08-14

Problem 1

(A) A put option gives the holder the right, but not the obligation, to sell an underlying asset at a predetermined price (the strike price) within a specified time period. A European put option can only be exercised at the expiration date, while an American put option can be exercised at any time up to and including the expiration date.

(B) The value of a call option is non-increasing in the strike price. This means that increasing the strike price cannot increase the value of the call option. The reason is that a higher strike price makes the right to buy the underlying asset less attractive, since the holder must pay more to purchase the asset if the option is exercised.

In many standard models, such as the Black–Scholes model, the value is strictly decreasing in the strike price. This is because there is a positive probability that the underlying asset price at maturity lies between any two different strike prices, making the option with the lower strike price more valuable.

Problem 2

(A) Since the bond is a zero coupon bond, its value at $t = 0$ is given by

$$P_0 = Fe^{-rT}$$

where $F = 1$ and $T = 5$. Hence, the current value of the bond is

$$P_0 = e^{-0.03 \cdot 5} = 0.8607.$$

To find the time t when the bond price P_t equals 0.95, we solve

$$0.95 = P_t = e^{-0.03(T-t)}$$

where $T = 5$. Taking logarithms on both sides gives

$$\ln(0.95) = -0.03(5 - t)$$

and therefore

$$5 - t = \frac{-\ln(0.95)}{0.03}$$

so that

$$t = 5 - \frac{-\ln(0.95)}{0.03} = 3.29.$$

Hence, the bond will be worth 0.95 approximately 3.29 years after time $t = 0$.

(B) The value of the bond P_0 is the present value of all future coupon payments and the face value:

$$P_0 = 8e^{-0.03\cdot 0.5} + 8e^{-0.03\cdot 1} + 8e^{-0.03\cdot 1.5} + (100+8)e^{-0.03\cdot 2} = 125.0030$$

Problem 3

(Based on Capinski & Zastawniak around p. 227.) The task is to find the value VaR that solves the equation

$$P\left(5S(0)e^r - 5S(1) < \text{VaR}\right) = 0.95.$$

The random variables $S(1)$ and $S(0)e^{\mu+\sigma Z}$ (where $Z \sim N(0,1)$) have the same distribution, which implies that the equation above is equivalent to

$$\begin{aligned}
&P\left(5S(1) > 5S(0)e^r - \text{VaR}\right) = 0.95,\\
\iff &P\left(e^{\mu+\sigma Z} > e^r - \frac{\text{VaR}}{5S(0)}\right) = 0.95,\\
\iff &P\left(\mu + \sigma Z > \ln\left(e^r - \frac{\text{VaR}}{5S(0)}\right)\right) = 0.95,\\
\iff &P\left(Z > \frac{\ln\left(e^r - \frac{\text{VaR}}{5S(0)}\right) - \mu}{\sigma}\right) = 0.95,\\
\iff 1 - &P\left(Z < \frac{\ln\left(e^r - \frac{\text{VaR}}{5S(0)}\right) - \mu}{\sigma}\right) = 0.95,\\
\iff &P\left(Z < \frac{\ln\left(e^r - \frac{\text{VaR}}{5S(0)}\right) - \mu}{\sigma}\right) = 0.05.
\end{aligned}$$

Recalling the value for the standard normal quantile $N^{-1}(0.05) \approx -1.645$, yields

$$\frac{\ln\left(e^r - \frac{\text{VaR}}{5S(0)}\right) - \mu}{\sigma} = -1.645.$$

Plugging in the values

$$r = 0.04, \qquad \mu = 0.12, \qquad \sigma = 0.25, \qquad S(0) = 20.$$

and solving for VaR gives

$$\ln\left(e^{0.04} - \frac{\text{VaR}}{100}\right) = 0.12 - 1.645 \cdot 0.25 = -0.29125, \Rightarrow$$

$$e^{0.04} - \frac{\text{VaR}}{100} = e^{-0.29125} = 0.7473, \Rightarrow$$

$$\frac{\text{VaR}}{100} = e^{0.04} - 0.7473 = 0.2935,$$

so that

$$\text{VaR} \approx 29.35.$$

Problem 4

We first calculate the option values at time $t = 1$ using replication: After the dividend payment of 5 SEK at time $t = 1$, the possible stock prices are

$$S^{u,+} = 100(1 + U) - 5 = 110 - 5 = 105,$$

$$S^{d,+} = 100(1 + D) - 5 = 90 - 5 = 85.$$

The possible stock prices at time $t = 2$ are therefore

$$S^{uu} = 105(1 + U) = 115.5,$$

$$S^{ud} = 105(1 + D) = 94.5,$$

$$S^{du} = 85(1 + U) = 93.5,$$

$$S^{dd} = 85(1 + D) = 76.5.$$

The terminal values of the European call option are thus (use that the strike is $X = 100$)

$$C_E^{uu} = (S^{uu} - X)^+ = 15.5,$$

$$C_E^{ud} = (S^{ud} - X)^+ = 0,$$

$$C_E^{du} = (S^{du} - X)^+ = 0,$$

$$C_E^{dd} = (S^{dd} - X)^+ = 0.$$

The replicating portfolio formed at time $t = 1$ in case the stock price at time 1 (after the dividend is paid) is $S^{u,+}$ should satisfy the replication equations:

$$x^u(2)S^{uu} + (1 + R)y^u(2) = C_E^{uu},$$

and

$$x^u(2)S^{ud} + (1 + R)y^u(2) = C_E^{ud}.$$

which yields that the number of shares is

$$x^u(2) = \frac{C_E^{uu} - C_E^{ud}}{S^{uu} - S^{ud}} = \frac{15.5 - 0}{115.5 - 94.5} = 0.7381.$$

and the amount invested in the risk-free asset is (recall $R = 0$)

$$y^u(2) = \frac{C_E^{ud} - x^u(2)S^{ud}}{1 + R} = \frac{-0.7381 \cdot 94.5}{1} = -69.75.$$

Hence, the corresponding option price is

$$C_E^u = x^u(2)S^{u,+} + y^u(2) = 0.7381 \cdot 105 - 69.75045 = 7.7500$$

It is directly seen that

$$C_E^d = 0$$

since if the stock price at time 1 (after the dividend is paid) is $S^{d,+}$ then the option will always yield zero payoff at time 2 (since $C_E^{du} = C_E^{dd} = 0$).

Now note that the replicating portfolio formed at time $t = 0$ should satisfy the replication equations:

$$x(1)(S^{u.+} + 5) + (1 + R)y(1) = C_E^u,$$

$$x(1)(S^{d.+} + 5) + (1 + R)y(1) = C_E^d.$$

The replicating portfolio formed at time 0 is therefore given by

$$x(1) = \frac{C_E^u - C_E^d}{S^{u.+} - S^{d.+}} = \frac{7.7500 - 0}{105 - 85} = 0.3875$$

and

$$y(1) = \frac{C_E^d - x(1)(S^{d.+} + 5)}{1 + R} = -34.8750.$$

The current value of the option is therefore

$$C_E(0) = x(1)S(0) + y(1) = 3.8750.$$

Problem 5

This is based on Capinski & Zastawniak around p. 84-87. The Capital Asset Pricing Model (CAPM) states that

$$\mu_i = R + \beta_i(\mu_M - R).$$

(A) Here we have

$$\beta = 1.2.$$

Thus, (using also the other values provided in the exam question) we find using the CAPM formula the expected return

$$\mu = 0.02 + 1.2 \cdot (0.05 - 0.02) = 0.056 = 5.6\%.$$

(B) The value invested in company A is

$$100S_A(0) = 100 \cdot 1 = 100,$$

and the value invested in company B is

$$50S_B(0) = 50 \cdot 1 = 50.$$

The total value of the portfolio is

$$100 + 50 = 150.$$

and the portfolio weights are therefore

$$w_A = \frac{100}{150} = \frac{2}{3}, \text{ and } w_B = \frac{50}{150} = \frac{1}{3}.$$

The beta of the portfolio is the weighted average of the individual betas (Capinski & Zastawniak p. 87):

$$\beta_V = w_A\beta_A + w_B\beta_B.$$

This means that

$$\beta_V = \frac{2}{3} \cdot 1.2 + \frac{1}{3} \cdot 1.1 = \frac{7}{6}.$$

Using the CAPM formula for the portfolio, we obtain that the expected return of the portfolio is

$$\mu_V = 0.02 + \frac{7}{6} \cdot (0.05 - 0.02) = 0.055 = 5.5\%.$$