Topology, Fall 2025

Gregory Arone

Homework exercise #6

Due: Thursday Nov 27, 2025

Instructions: We strongly prefer that you typeset your answers with Latex or similar. Handwritten assignments will be graded only if easy to read.

You may work together on the assignment, but write your own solution and do not copy. You also are welcome to consult the instructors, after you made a decent effort on a problem. Tell us what you have tried, and we will be happy to provide some direction if needed.

Please, please, do not seek inappropriate help on the internet or from ChatGPT and the like.

1. Let $T^2 = S^1 \times S^1$ be the torus and let $X = T^2 \vee S^2$.
   (a) Describe the fundamental group of X and the universal cover of X. For the latter, a clear picture accompanied with a short and clear explanation should suffice. (10 points)
   (b) How many pairwise non-isomorphic connected covers of X of degree 3 are there? (20 points)
2. Let S^2 be the unit sphere. Define the following equivalence relation on S^2:
   (a) $(0,0,1) \sim (0,0,-1)$, and
   (b) For all (x,y) that satisfy $x^2+y^2=1$, $(x,y,0) \sim (-x,-y,0)$.

   Let X be the quotient of S^2 by this equivalence relation. Calculate $\pi_1(X)$. Here by "calculate" we mean "give an explicit presentation of". Is $\pi_1(X)$ finite? Is it abelian? (20 points)