

### HOMEWORK SET 3

**HW1.** Prove that every finite based CW complex  $X$  has the following property: *for every based CW complex  $Y$ , the maps  $[\Sigma^n X, \Sigma^n Y] \rightarrow [\Sigma^{n+1} X, \Sigma^{n+1} Y]$  are isomorphisms for all sufficiently large  $n$ .*

(Hint: Prove the result by induction on the number of cells. Use the Puppe sequence associated to  $S^k \xrightarrow{\phi} X \rightarrow X \cup_{\phi} e^{k+1}$ , and the five lemma.)

**HW2.** Calculate the cohomology ring of the homotopy fiber of the map  $\mathbb{CP}^n \rightarrow \mathbb{CP}^{n+1}$ .

*Deadline: 2025-11-06. If you have used any resources outside the course literature/lecture notes, please indicate this in your solution. Similarly if you have discussed the problems with another student, or used AI. Hand in your solutions by e-mail to: dan.petersen@math.su.se*