LCA or No LCA:
A short story about simplifying networks

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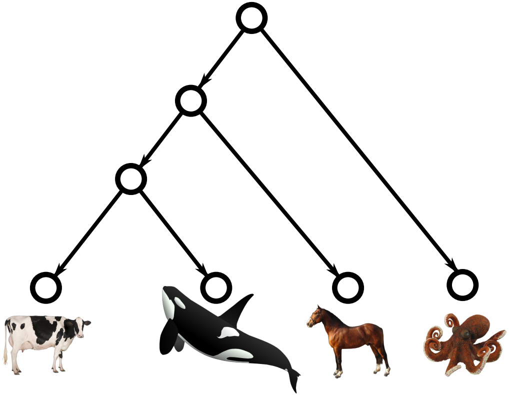
Faculty of Science
Stockholm University

40th TBI Winterseminar in Bled, 2025

Evolutionary Histories . . .



Classical Tree-like Relationships

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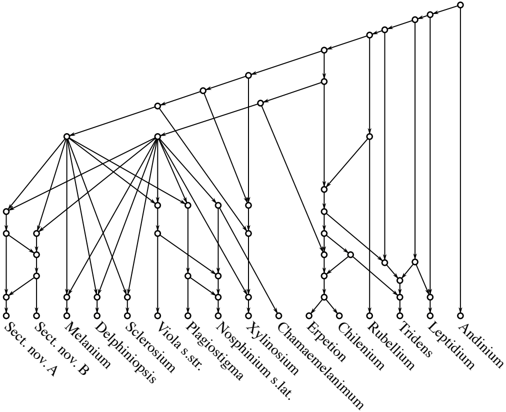

Network of some *Viola* plants*

Well in these inconspicuous plants . . .

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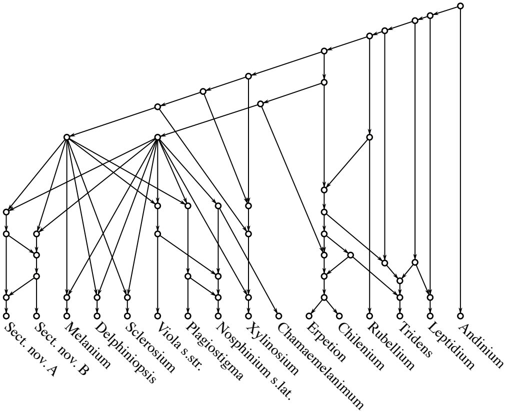

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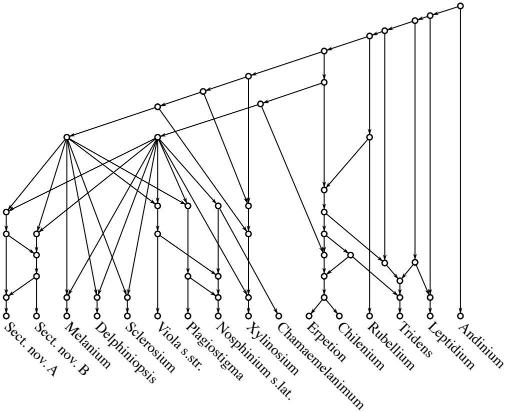

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(when plants decide monogamy is overrated)

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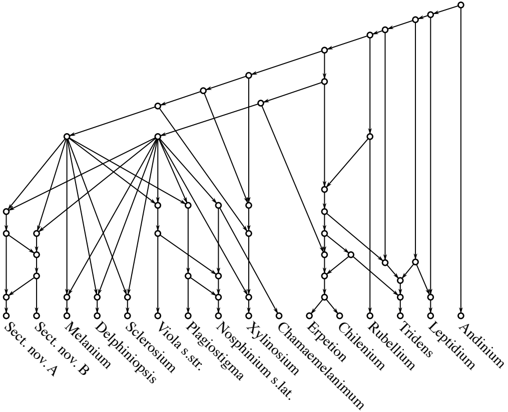

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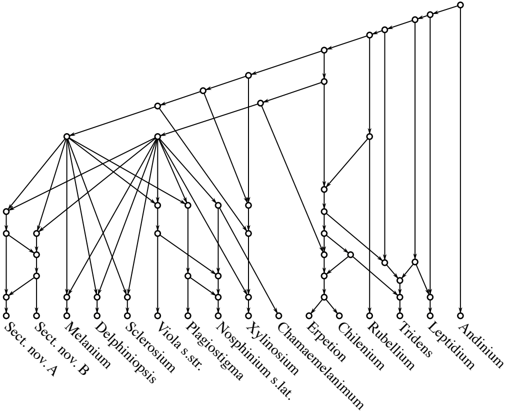

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and many more
(now the weired sexual preferences of these cute plants go beyond my limits of imagination)

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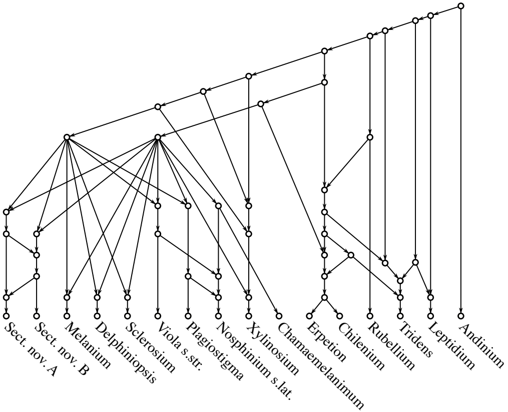
Mechanisms like HGT or hybridiations cannot be fully captured by trees
$\Longrightarrow$ evolutionary history is network-like



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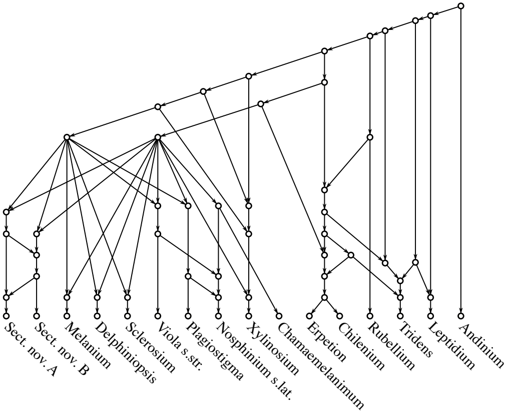

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Start with observable data
= genomic sequences of extant taxa
= set $L(N)$ of leaves in network N we want be infer

Based on observable data infer the network $N^{\ddagger}$

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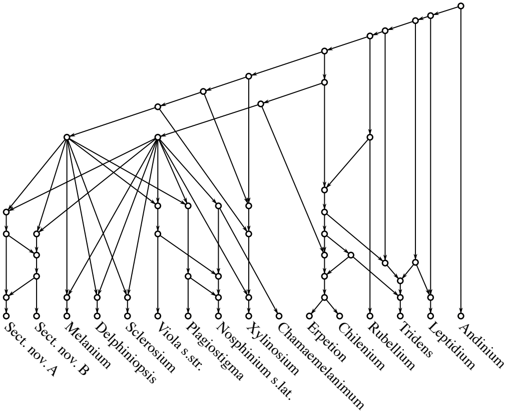

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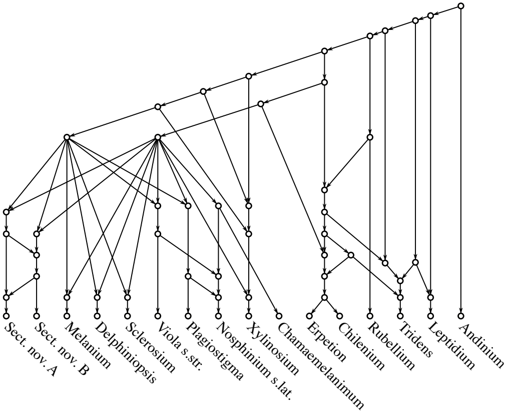

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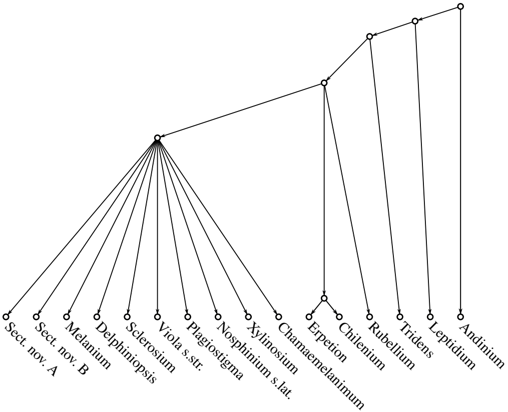

Lowest Stable Ancestor (LSA) tree†
to show "trend of evolution"

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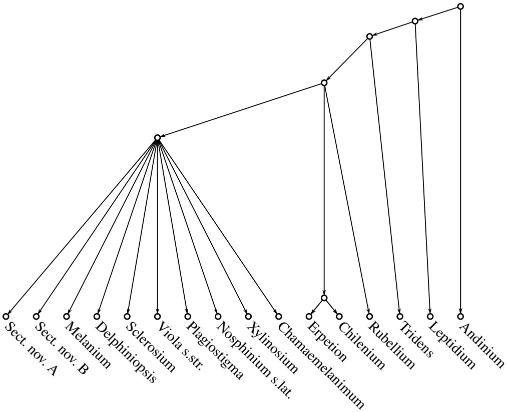

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The method we propose aims to simplify by eliminating vertices that are not supported by "observable data"

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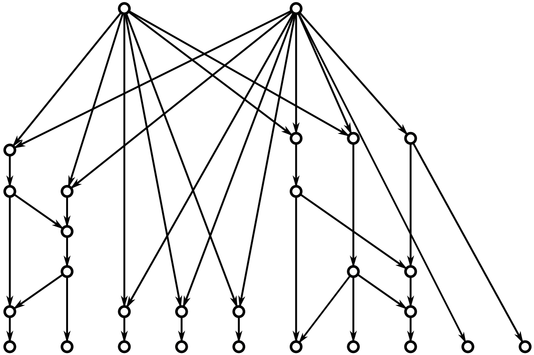

DAG = Directed Acyclic Graph.

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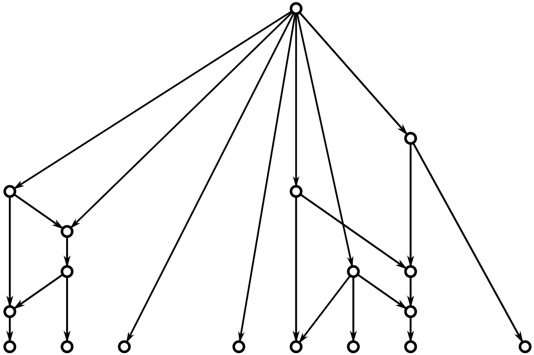

DAG = Directed Acyclic Graph.

If a DAG has unique source (=root), it is a network

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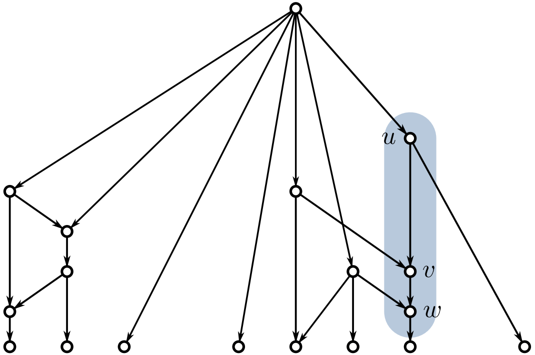

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Denoted $w \preceq u$.

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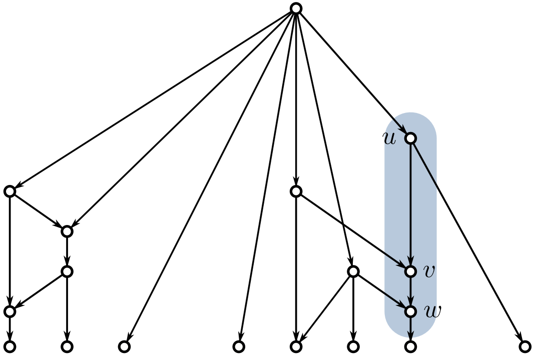

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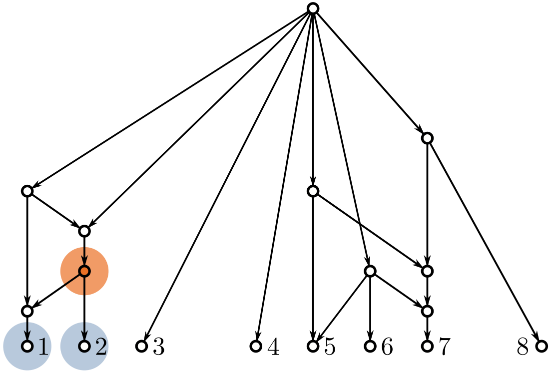

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LCA$(\{1,2\})$

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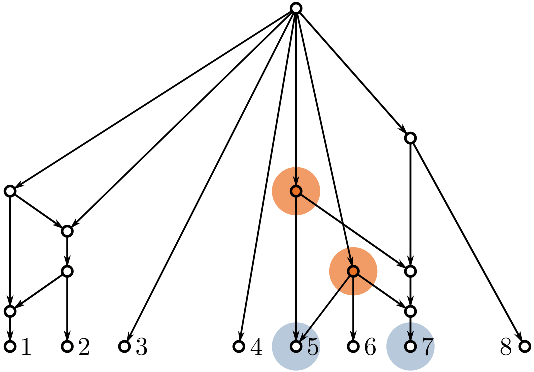

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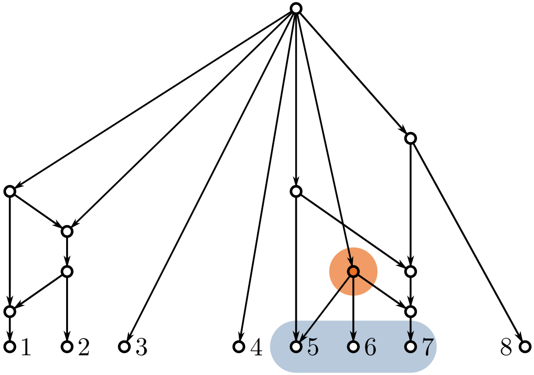

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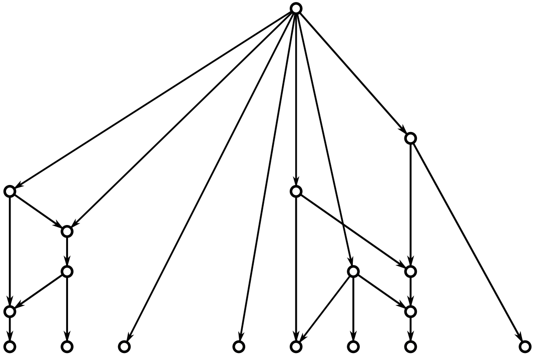

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What does "supported" mean?

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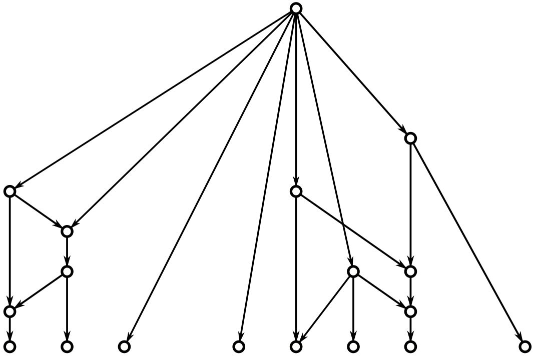

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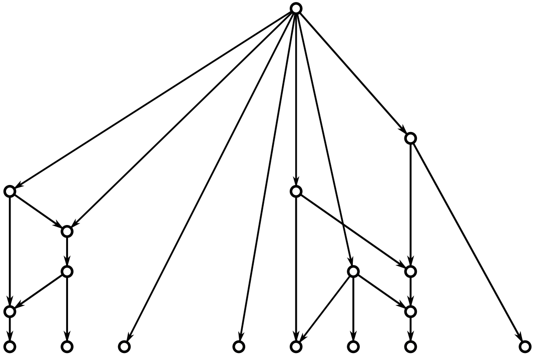

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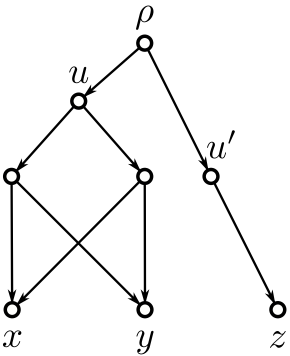

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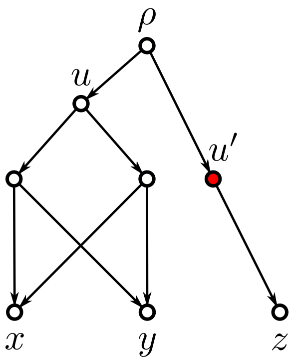

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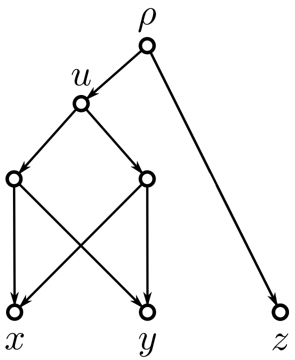

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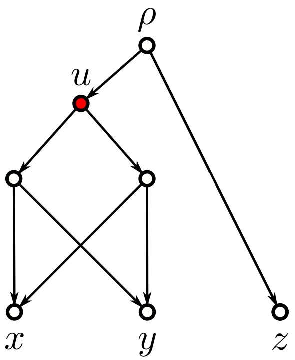

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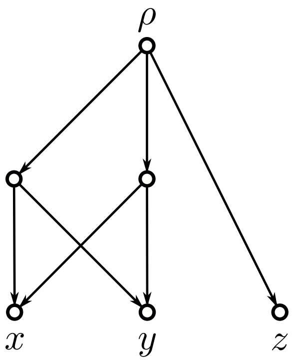

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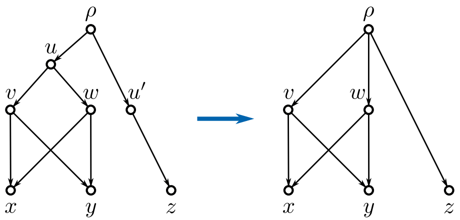

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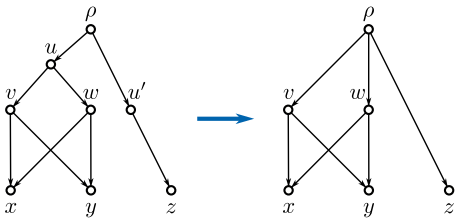

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This begs several questions:

- How to determine unsupported vertices and how to get rid of them?
  Testing all $A \subseteq L(G)$ and check if $v \in \mathrm{LCA}(A)$ is not an option!
- Can we characterize LCA-RELEVANT DAGs?

Results: Characterization

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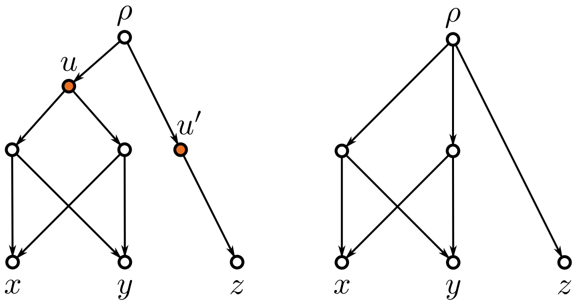

not LCA-RELEVANT LCA-RELEVANT

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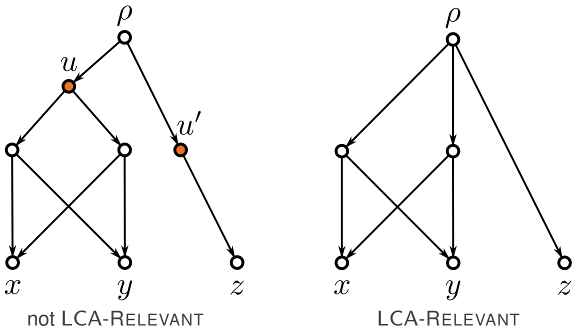

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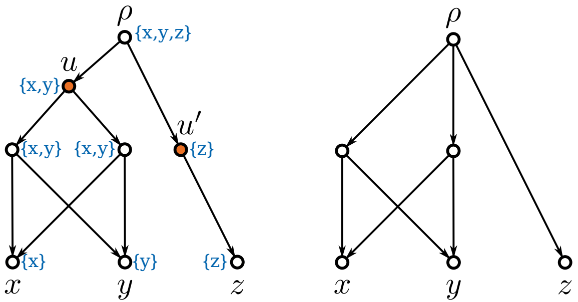

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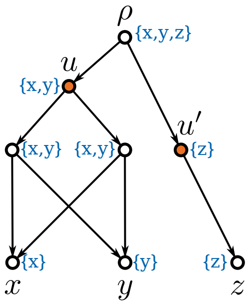

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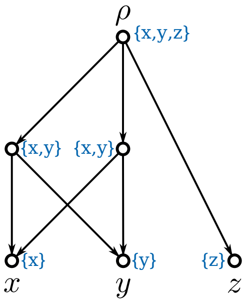

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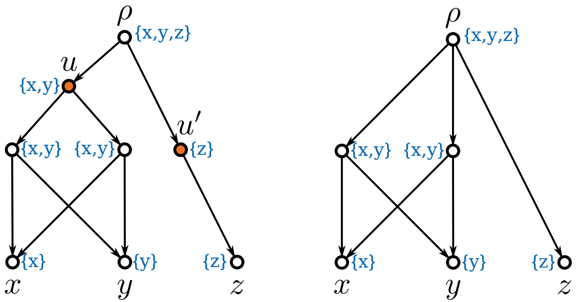
*All non-*LCA*-vertices can be determined in polynomial-time.*



not LCA-RELEVANT LCA-RELEVANT

Results: Characterization

Questions:

- How to determine unsupported vertices and how to get rid of them ?
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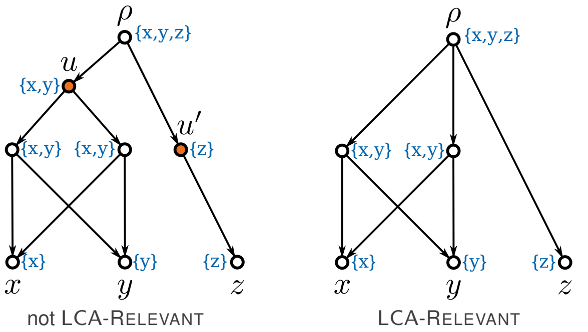

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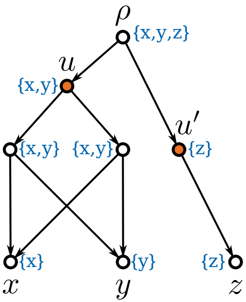

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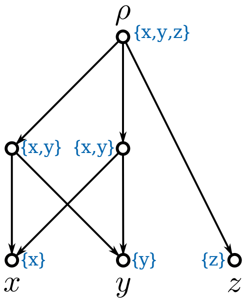

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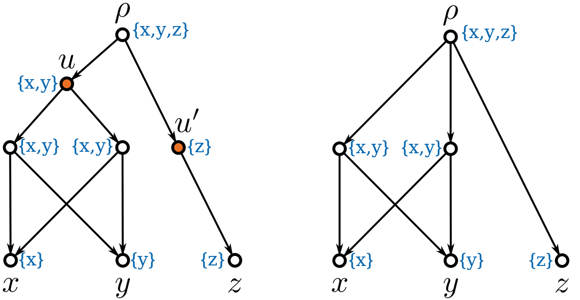

not LCA-RELEVANT   LCA-RELEVANT

Results: The $\ominus$-operator

If v is a vertex of G, then $G \ominus v$ is obtained by adding an edge from each parent of v to each child of v, and then removing v.

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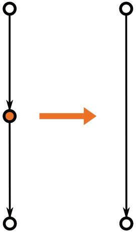

The $\ominus$-operator generalizes the concept of “vertex-suppression”

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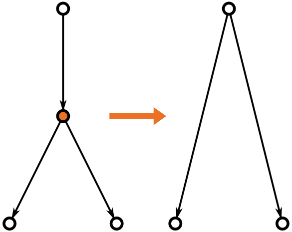

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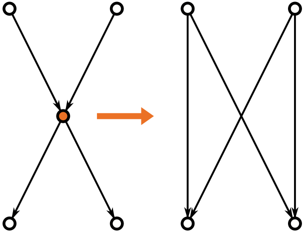

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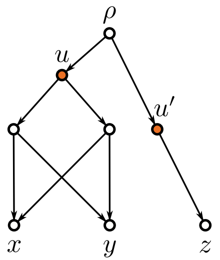

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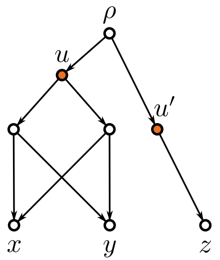

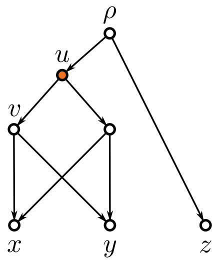

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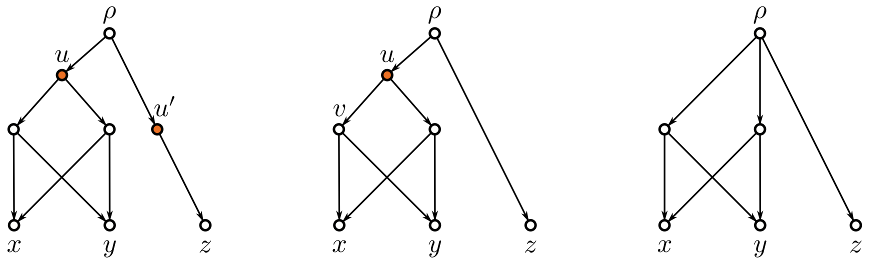

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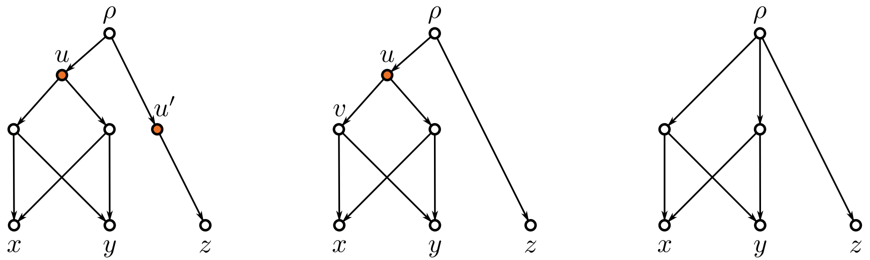

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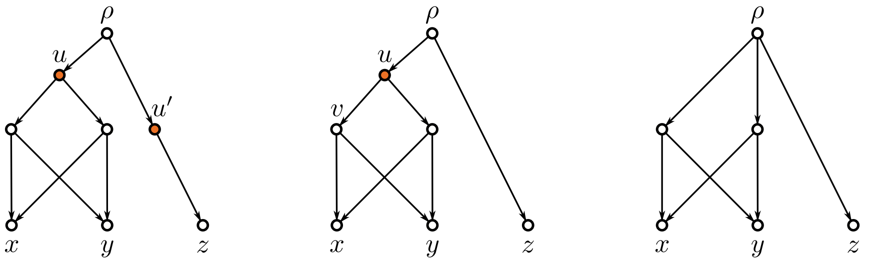

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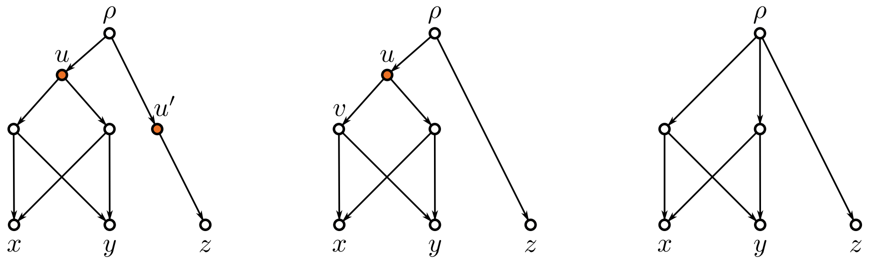

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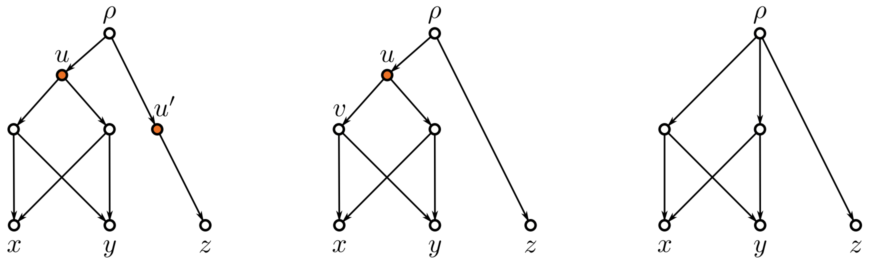

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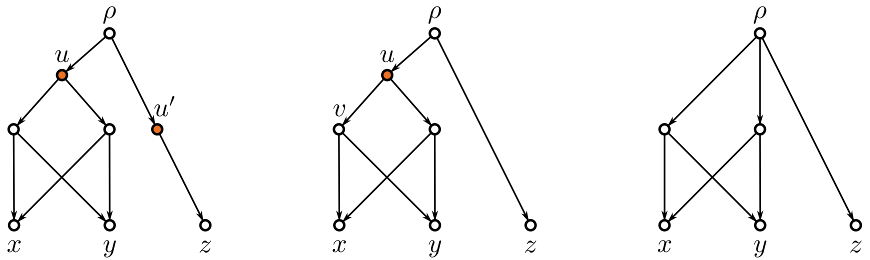

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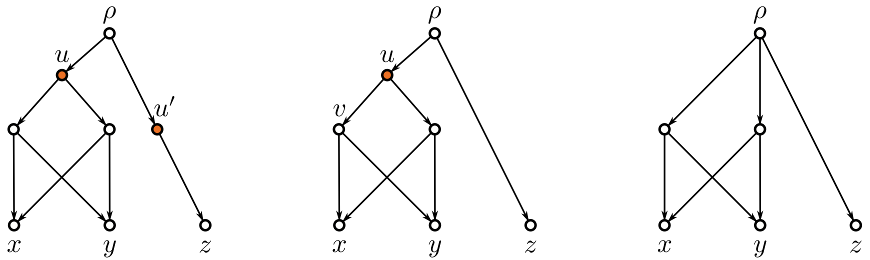

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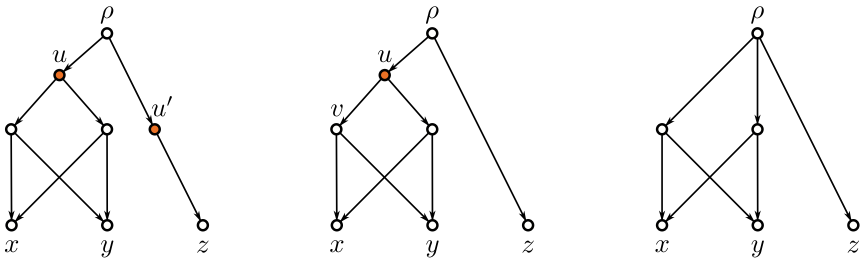

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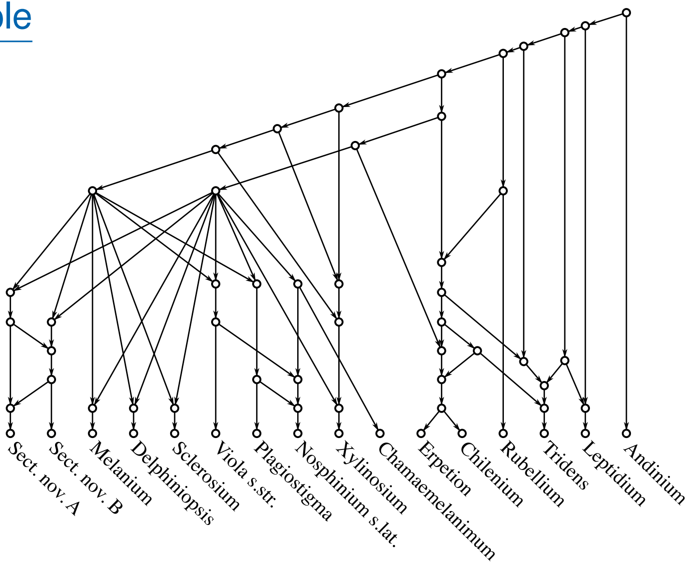
Moreover, W is the minimum-sized set for which $G \ominus W$ is LCA-Relevant *and $G \ominus W$ can be computed in polynomial time.*

Results: An example

Back to our Plant-Tinder-Elites . . .



Network N

Results: An example

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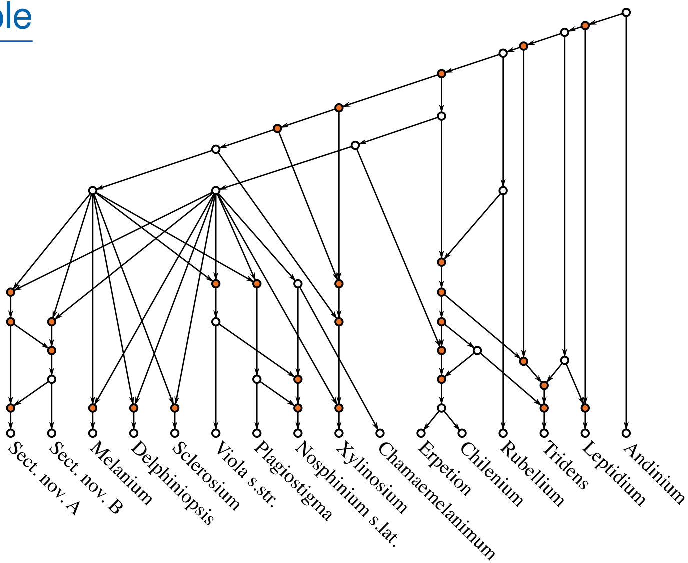
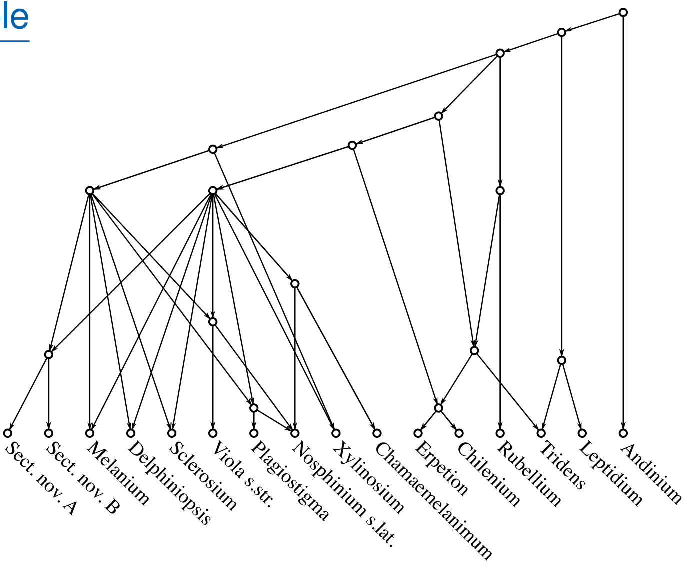


Network N with highlighted *non*-LCA vertices

Results: An example

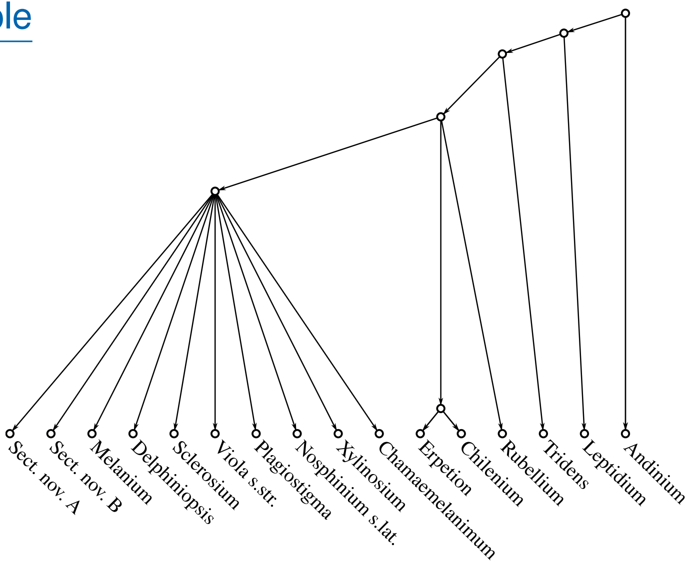
Back to our Plant-Tinder-Elites . . .



The unique LCA-RELEVANT version $N \ominus W$ of N

Results: An example

Back to our Plant-Tinder-Elites . . .



LSA-tree of N to show “trend of evolution”

Results: Let's strengthen LCA-RELEVANT

A DAG G is LCA$^{\mathrm{u}}$-RELEVANT if each vertex v is the unique LCA for some $A \subseteq L(G)$, i.e., LCA$(A) = \{v\}$

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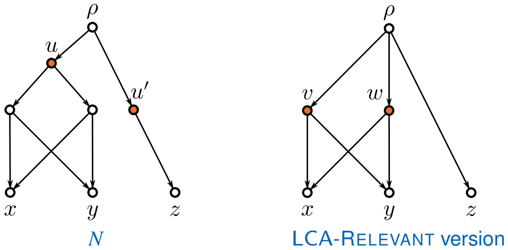

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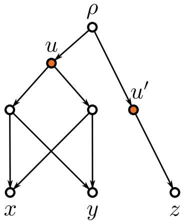

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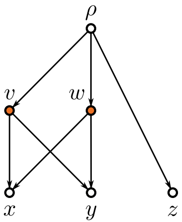
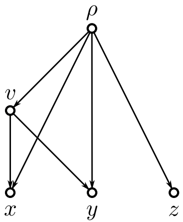


LCA-RELEVANT version



an LCA$^{\text{u}}$-RELEVANT version

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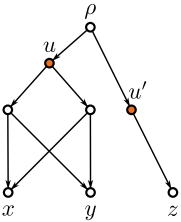

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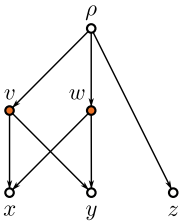

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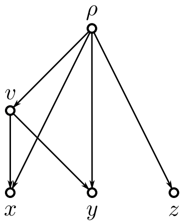

an LCA$^{\mathrm{u}}$-RELEVANT version

Theorem.
For a network or DAG G the following are equivalent:

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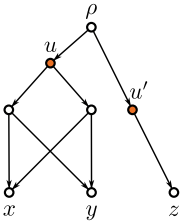

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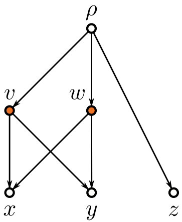

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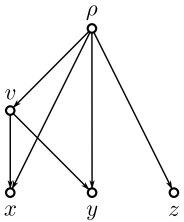

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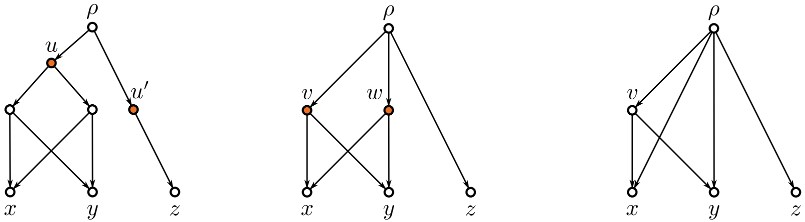

N LCA-RELEVANT version an LCA$^{\mathrm{u}}$-RELEVANT version

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For a network or DAG G the following are equivalent:

- *G is* LCA$^{\mathrm{u}}$-RELEVANT
- *G is* LCA-RELEVANT *and satisfies (PCC)*

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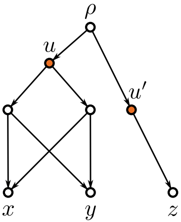

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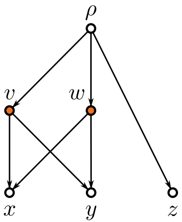
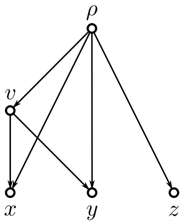


LCA-Relevant version

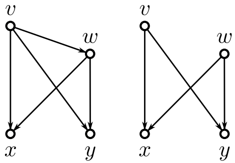


an LCA^{u}-Relevant version

Theorem.
For a network or DAG G the following are equivalent:

- *G is LCA^{u}-Relevant*
- *G is LCA-Relevant and satisfies (PCC)*

A DAG satisfies (PCC): there is a vw-path if and only if $\mathtt{C}(v)$ and $\mathtt{C}(w)$ are comparable w.r.t $\subseteq$.



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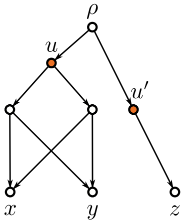

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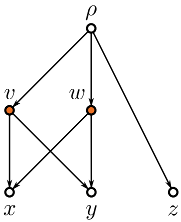
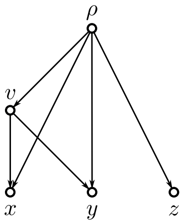


LCA-RELEVANT version



an LCA^{u}-RELEVANT version

Theorem.
For a network or DAG G the following are equivalent:

- *G is LCA^{u}-RELEVANT*
- *G is LCA-RELEVANT and satisfies (PCC)*
- *If you remove all shortcuts from G, what you obtain is isomorphic to the Hasse diagram of $\mathfrak{C}_G$*

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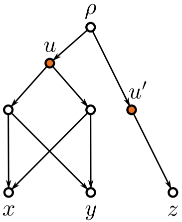

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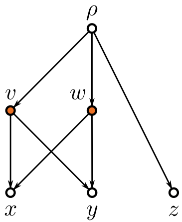
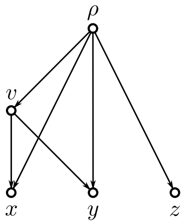


LCA-RELEVANT version



an LCA^{u}-RELEVANT version

Theorem.
For a network or DAG G the following are equivalent:

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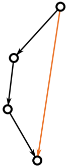
A shortcut is an edge (u, v) for which there is some other uv-path in G.



Results: Let's strengthen LCA-RELEVANT

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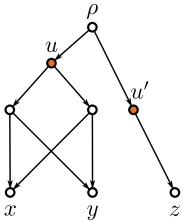

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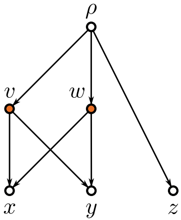

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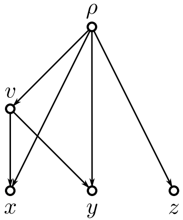

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$\{x, y, z, w\}$

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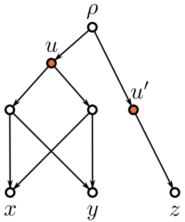

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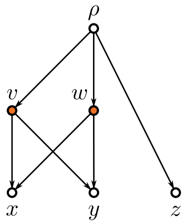

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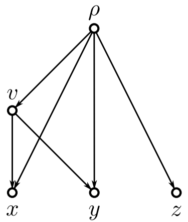

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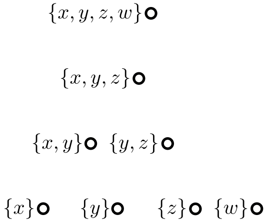

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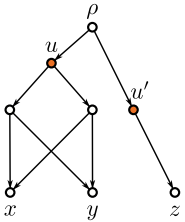

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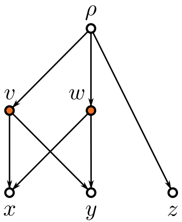

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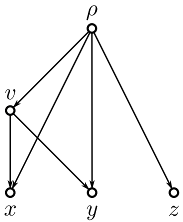

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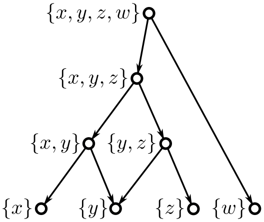

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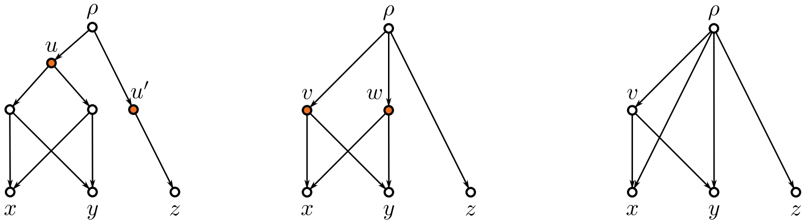

N | LCA-RELEVANT version | an LCA$^{\mathrm{u}}$-RELEVANT version

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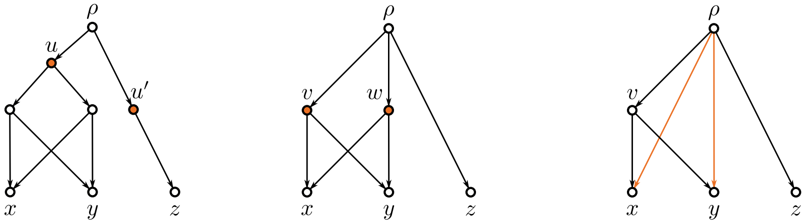

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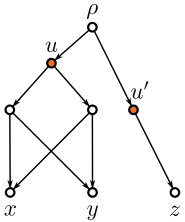

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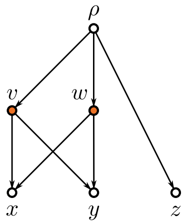

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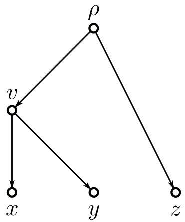

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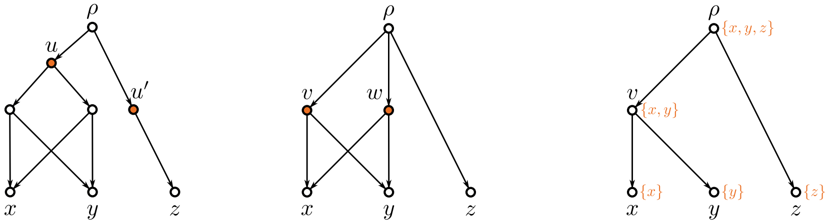

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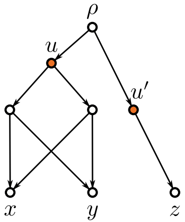

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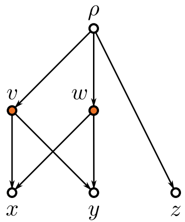

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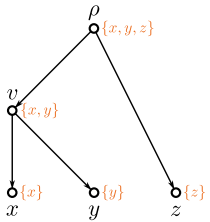

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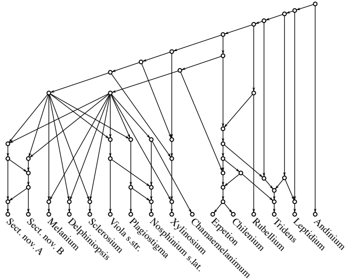
Computing an LCA$^{\mathrm{u}}$-RELEVANT *version of any DAG G can be done in polynomial-time*

Results: An example

Back to our Plant-Tinder-Elites . . .



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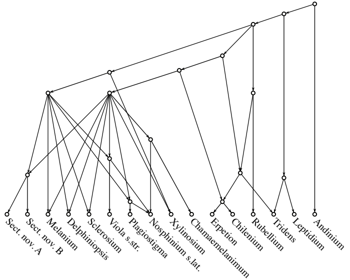
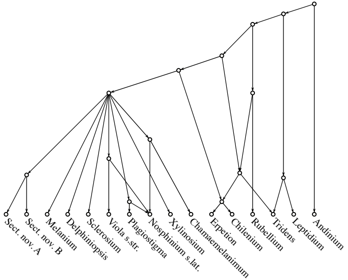


the LCA-RELEVANT version of N



an LCA^{u}-RELEVANT version of N (all shortcuts removed)

All these simplifications can be computed in polynomial-time and are implemented as python-tool `LCA-SimpliDAG`

Lindeberg and Hellmuth, Simplifying and Characterizing DAGs and Phylogenetic Networks via Least Common Ancestor Constraints, Bulletin of Math. Bio. (submitted), 2024

https://github.com/AnnaLindeberg/LeastCommonAncestor-relevant.git

Other results and open problems

- Generally worked with a generalization: a DAG is $\mathscr{I}$-LCA-RELEVANT if for every vertex $v \in V(G)$ there is some $A \subseteq L(G)$ s.t. $|A| \in \mathscr{I}$ and $v \in \mathrm{LCA}(A)$.

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- Is it too much to require vertices to be LCA's of leaves?

  Recursive Def:

  Every leaf in G is pertinent.

  A vertex is pertinent if it is the LCA of pertinent vertices.

  What is the structure of DAGs in which all vertices are pertinent and how to determine such vertices?

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Joint work with

Anna Lindeberg

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Thank You!