

Problem 1: Algebraic expressions

Simplify the following algebraic expressions completely:

$$\begin{aligned} \text{(a)} \quad & \frac{x^2 - 4}{x^2 - 4x + 4} \cdot \frac{2x - 4}{x^2 + 2x} \\ \text{(b)} \quad & \frac{x - 1}{x + 1} - \frac{x + 1}{x - 1} + \frac{4x}{x^2 - 1} \end{aligned}$$

Problem 2: Inequalities

Find the set of all real numbers x that satisfy the following inequalities. Express your final answer using interval notation.

$$\begin{aligned} \text{(a)} \quad & (x^2 - 3x - 10)e^{2x} \leq 0 \\ \text{(b)} \quad & \frac{2x - 3}{x + 2} > 1 \end{aligned}$$

Problem 3: Absolute value

Solve the following absolute value inequality by removing the absolute value bars and considering the two algebraic branches separately:

$$|3x - 6| \geq 9$$

Problem 4: Taylor Polynomial

Consider the function $f(x) = \ln(2x - 1) + x^3$.

- (a) Find the standard Taylor polynomial of degree 3, $P_3(x)$, associated with $f(x)$ centered around the point $a = 1$.
- (b) Give an approximation of $f(1)$.

Problem 5: Implicit derivation

Consider the curve implicitly defined by the equation:

$$y^3 + x^2y + x^3 - 3 = 0$$

- (a) Verify that the point $(1, 1)$ lies on this curve.
- (b) Compute the slope of the tangent line $\frac{dy}{dx}$ at $(1, 1)$ using implicit differentiation.

- (c) Find the degree 2 (quadratic) Taylor polynomial $p_2(x)$ for the implicit function $y(x)$ centered around $a = 1$ by calculating $\frac{d^2y}{dx^2}$ at $(1, 1)$.

Problem 6: Geometric sums in economics

A Swedish clean-energy startup expects to generate a stable, recurring cash payout of $A = 50,000$ SEK at the end of every year. Due to regional market growth, this payout is projected to expand by a compound growth rate of 4% each year (meaning the Year 1 payment is A , the Year 2 payment is $A(1.04)$, the Year 3 payment is $A(1.04)^2$, etc.).

An investor wants to buy a financial stake in the company that entitles them to the first 20 years of payouts. If the baseline market discount interest rate is 6% annually:

- Write out the mathematical summation representing the Total Present Value (PV) of this 20-year cash stream.
- Use the finite geometric sum formula to calculate the exact total present value of this investment.

Problem 7: Convergence of geometric series

Consider the infinite series of functions defined by:

$$S(x) = \sum_{n=0}^{\infty} \left(\frac{2x}{x^2 + 1} \right)^n$$

Determine the exact set of real numbers x for which this series converges. Express your final answer using interval notation.

(Hint: Identify this as a geometric series of functions where the common ratio $r(x)$ is a composite function.)

Problem 8: Continuity

A bulk supplier of specialized manufacturing components structures its total revenue function $R(q)$, in thousands of SEK, based on the order quantity q (where $q > 0$). Due to a bulk discount policy at $q = 50$ units, the function is defined piecewise:

$$R(q) = \begin{cases} 2q + \frac{q^2}{50} & \text{if } 0 < q \leq 50 \\ \frac{a \cdot \ln(q-49) + 180}{\sqrt{q-14}} & \text{if } q > 50 \end{cases}$$

Determine the exact value of the constant parameter a that ensures the total revenue function $R(q)$ is **continuous** at the pricing threshold $q = 50$.

Problem 9: Limits

Evaluate the following limits using l'Hôpital's rule. Make sure to explicitly verify that the expressions satisfy the required indeterminate criteria ($\frac{0}{0}$ or $\frac{\infty}{\infty}$) before differentiating.

$$(a) \quad \lim_{x \rightarrow 1} \frac{\ln(x) - x + 1}{x^2 - 2x + 1}$$

$$(b) \quad \lim_{x \rightarrow 0^+} \frac{\ln(e^{3x} - 1)}{\ln(x)}$$

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