MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Instructor: Benjamin Nill

Combinatorics II (Omtentamen)
February 15, 2014
9:00 - 14:00

The exams will be returned on Monday, February 24, at 15:00 in Hus 6, Room 110; afterwards you will be able to pick them up from Katarina Ringels (Hus 6, Rum 204).

Complete and clear solutions must be given
except where explicitly stated otherwise.

No calculators allowed.
If you cannot simplify an expression any further, just leave it.

1. (2,5 points) At a wedding party a kid gets bored. It picks up the 50 nametags of the wedding guests (no two of the wedding guests have the same name) and puts them into 8 empty glasses (some of them can still be empty afterwards). In how many ways can this be done, if

(a) (1 point) the glasses look all different?
(b) (1,5 point) the glasses look the same?

2. (3 points) Let r be a positive integer. We define a_r as the number of (unordered) partitions of r into positive summands, where

- no summand is larger than at most 6, and
- each summand appears an even number of times, and
- the numbers 1 and 3 must appear as summands, and
- 2 appears at most two times as a summand.

Write the generating function of a_r as a quotient of two polynomials.

Hint: Make sure to read the problem very carefully.

3. (4 points) Let a_n be the sequence that satisfies $a_0 = 1$, $a_1 = 17$, and for $n \geq 2$ the recursion relation

$$a_n - 6a_{n-1} + 9a_{n-2} = n3^n.$$

Give a closed formula for a_n.

Clearly present every step of your computation.

4. (3 points) Find the number of permutations of $\{1, \ldots, 10\}$

(a) (1,5 points) with at least one fixpoint.
(b) (1,5 points) with precisely one fixpoint.

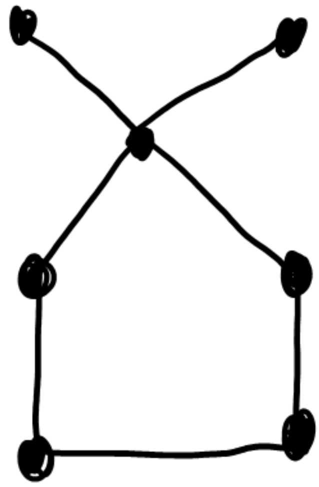
Remember that a fixpoint of a permutation is an element that is not changed by the permutation.

You have to show your reasoning.

5. (3 points) Let G be a (loop-free, undirected) graph with 24 vertices and 45 edges. Prove that if G is bipartite, then G cannot be planar.

Hint: How 'small' can a region of a bipartite planar graph be?

6. (3 points) Find the chromatic number *and* the chromatic polynomial of the following graph:

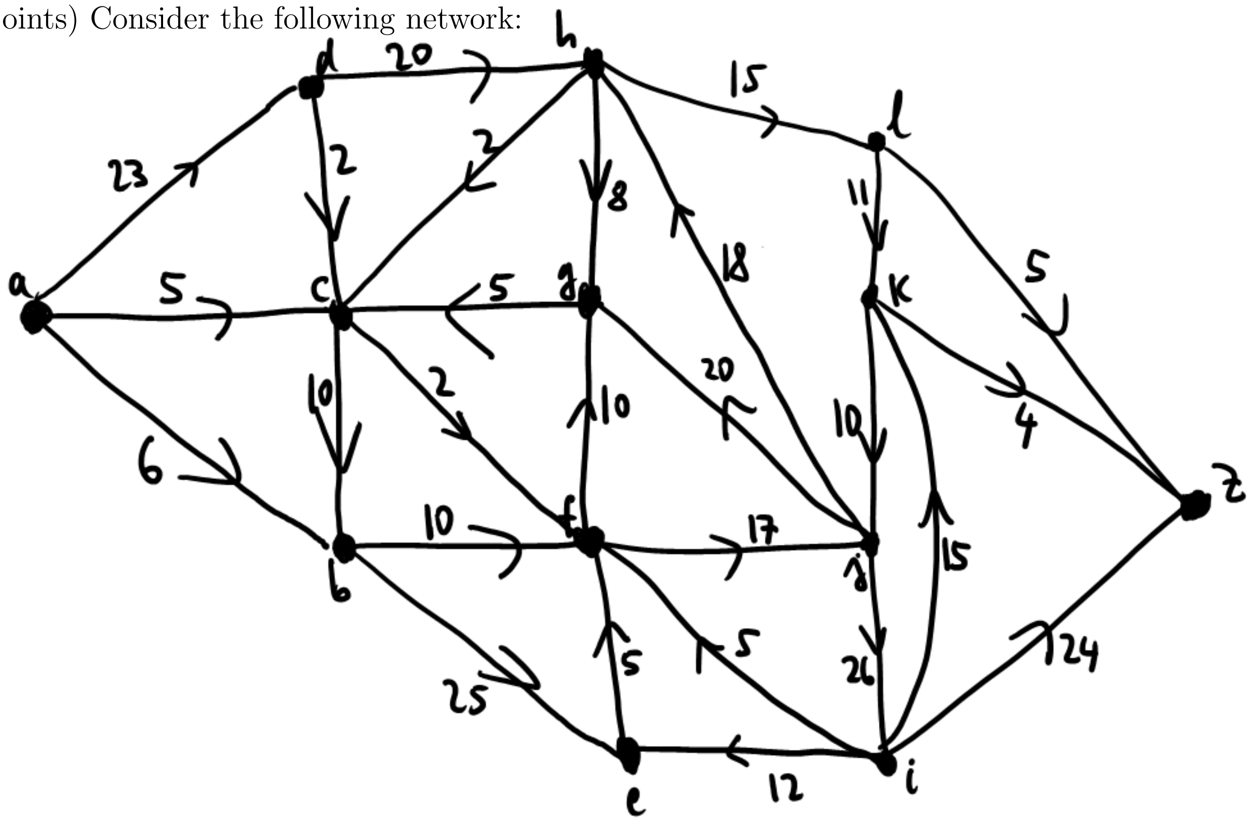


7. (2,5 points) At the international arctic research station it has become a ritual that every newcomer has to write down a new word into the station's logbook using only the letters in the word ARCTICA. For instance, RTAA is an allowed word, but not CAAAR. After how many visitors is it impossible to find a new word not yet written down?

As usual it is enough to write down an explicit formula, you don't have to evaluate it.

8. (2,5 points) In how many ways can one put 13 identical balls in 20 numbered boxes such that each of the first 5 boxes contain at least one ball, and each of the last 15 boxes contain at most 3 balls?

9. (6,5 points) Consider the following network:



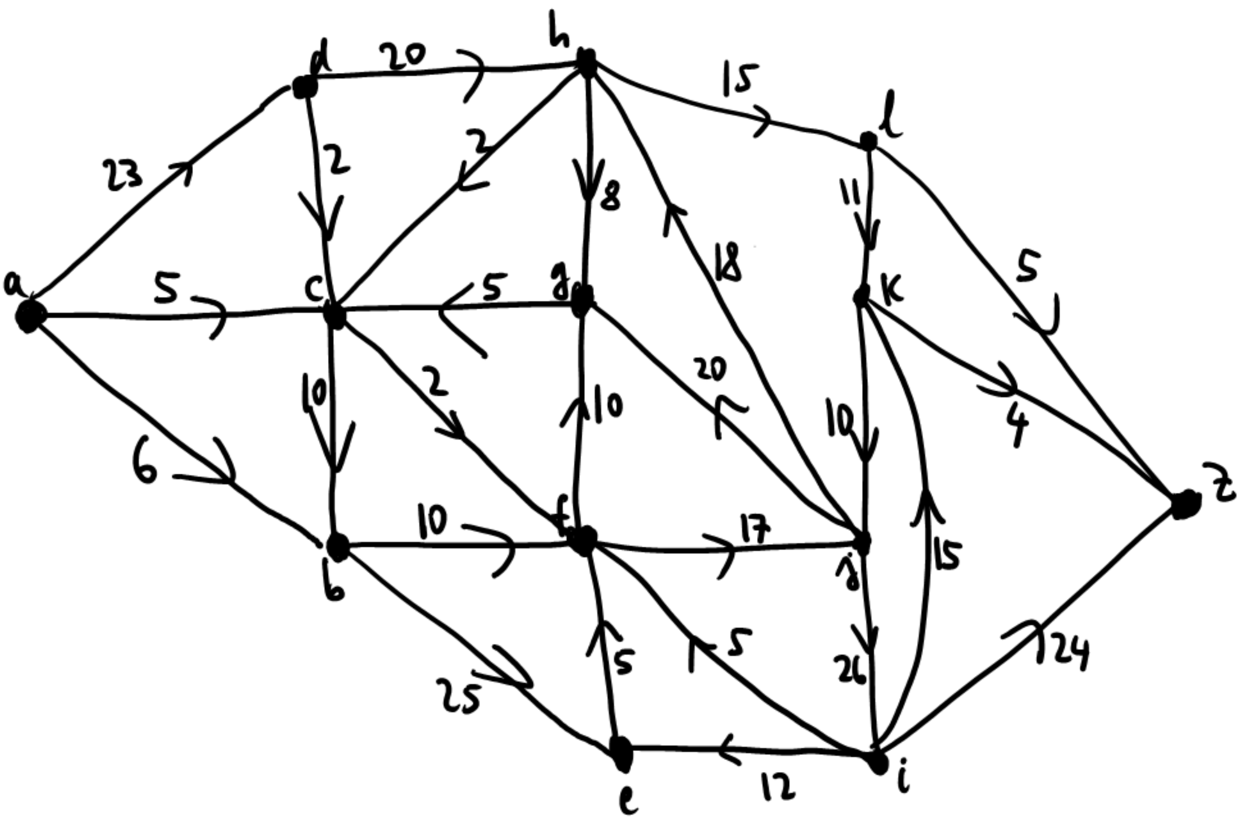
(a) (2 points) Give the shortest directed path from a to l. (I repeat: to l, not to z). Show your reasoning.

(b) (3 points) Give a flow with maximal flow value **ON THE NEXT PAGE**.

(c) (1,5 points) Give a cut with the minimal cut capacity **ON THE NEXT PAGE**.

*****HAND IN THIS PAGE*****

Enter the max flow you found right here into the network (next to the capacities):



The maximal flow value equals:

A cut set with minimal cut capacity is given as:

The minimal cut capacity equals: