

Fullständiga lösningar krävs, där inte annat uttryckligen anges. Complete solutions should be given, except where otherwise explicitly stated.

Hjälpmedel: Endast sedvanliga skriv- och ritverktyg. Pocket calculators, et cetera, are NOT allowed.

1. 170 balls are placed into 35 boxes, in such a manner that no box is left empty. Prove that then there are at least five boxes containing the same amount of balls. 4 p

2. 8 balls are placed into 3 boxes. At most one of the boxes may be left empty. In how many ways can you do this, if:
 - a) Both the balls and the boxes are unmarked (and thus may be treated as identical)? 1,5 p
 - b) Both the balls and the boxes are marked (with different labels)? 1,5 p
 - c) The boxes, but not the balls, are marked? 1,5 p
 - d) The balls, but not the boxes, are marked? 1,5 p

3. Determine the number of words of length
 - a) 4, 2 p
 - b) 10, or 2 p
 - c) 11, 2 pfrom the letters in the word **MATTEMATLAG**.

4. Determine the chromatic polynomial of the graph given in Figure 1 at the last page. 6 p

5. At present, there are 3 kinds of Swedish coins in circulation, with the values 1 SEK, 5 SEK or 10 SEK, respectively.
 - a) Exhibit the generating formal power series for the number of ways to pay an amount of k SEK in coins, as the quotient of two polynomials. 2 p
 - b) Calculate the number of ways to pay 43 SEK with coins. 3 p

Please, turn over!

6. In a forgotten valley in the mysterious mountainous regions of Central Europe, there lives a tribe, adhering to a faith they call “pantritarianism”. They claim that three is a holy number, that the whole universe is organised in triplets, both in large and small scale[skala]; and therefore, in everything they make, including scales[vågskålar]. Actually, instead of pairs of scales[balansvågar], as used by other people, their diligent and skilled artisans[flitiga och skickliga hantverkare] have constructed *triplets of scales*, where three objects or collections of objects are weighed against each others. A ‘triplet of scales’ consists of three bars, in rigid position[stela] to each others, and with a scale hanging from each end. The whole construction is hanging loosely in a rope, fixed at its centre of gravity[tyngdpunkt].

Thus, the scales will remain equally high, if the three scales are filled with the same weight. If not, either one will be lower than the other two, or one will be higher than the other two, or one will be lower and one higher than the third one, depending on whether some or all three scales contain different weights.

(The pantritarians point out that thus their construction has a much larger number of possible outcomes than a usual pair of scales has, and argue that this is a proof of the validity of their faith.)

The tribe uses an old kind of coins (called ‘ecus’) as money. Unhappily, there are some forgeries around, which only can be distinguished from the valid coins by their weights. The customary way to find the false coins is of course by means of triplets of scales. They have developed some general methods for finding forgeries quickly, in some often encountered situations.

- a) Provide a procedure for finding the false coin among $n \geq 3$ coins with as few weighings as possible, given that indeed exactly one of them is false, 4 p
- b) Give a logarithmic lower bound for the number of weighings needed to find both false coins among $n \geq 3$ ones, given that exactly one of the coins is too light, and one too heavy. (The two coins together weigh precisely as much as two valid coins.) 2 p
- c) Determine the actual maximal possible value of n in the situation of the preceding[föregående] subproblem, such that two weighings suffice. 2 p

The exams will be returned in my office (room 409, house 6, Kräftriket), 16.45 hours (a quarter to five P.M.), on Thursday, May 09, or by individual agreement.

Figure 1:

