Department of Mathematics STOCKHOLM UNIVERSITY Examiner: Paul Vaderlind	Examination in Mathematics for Economic and Statistical Analysis Master Program; 7,5 ECTS August, 21, 2012; time: 09:00–14:00

No calculators are allowed. Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

---

1. The expression $(x+1)^3y^2 + \ln(x+1)\ln y + e^{xy} = 2$ defines y as a function of x. Find the equation of the tangent line to this curve at the point $(x, y) = (0, 1)$, and find the point where this line crosses the x-axis.

2. Find the following limits:

a) $\lim_{x\to 5} \dfrac{x^3 - 5x}{\sqrt{5} - \sqrt{x}}$, b) $\lim_{x\to 2} \left(\dfrac{5x+2}{x^2-4} - \dfrac{3}{x-2}\right)$, c) $\lim_{x\to 0} \dfrac{1 + 2x - e^{2x}}{x\ln(x+1)}$

3. Find the numbers a and b such that the function $f(x) = \dfrac{ax^2 + bx + 1}{x - 1}$ has a local extreme at the point $f(2) = 3$. Is it a local max- or minimum point?

4. Find all stationary points for the function $f(x, y) = xe^{-(\frac{1}{2}x^2 + y^2 + 2y)}$ and determine whether they are maximum, minimum or saddle points.

5. a) Use the Cramer's rules to solve the system of equations

$$\begin{cases} 4x + 5y + 3z = 25 \\ \quad\;\; y + z = 5 \\ 2x + 3y + z = 13 \end{cases}$$

b) Given three matrices: $A = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 9 & 3 \\ 3 & 0 \end{pmatrix}$, solve the matrix-equation $A \cdot X + B = C$.

6. a) For which real numbers x is the serie $S = 1 + \dfrac{2}{3}\ln x + \dfrac{4}{9}\ln^2 x + \dfrac{8}{27}\ln^3 x + ...$ convergent?

b) Find x for which $S = \frac{3}{2}$.

7. Consider the function $f(x) = \dfrac{1}{\ln(x^2 - 1)}$.

a) For which x is the function $f(x)$ well-defined?

b) For which x is $f(x)$ increasing and for which x is $f(x)$ decreasing?

GOOD LUCK!

The papers will be handed out at 11.00 on Wednesday, September 5, 2012, in the room next to the Coffee Shop, house 5, and after that in room 208, house 6.